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SOLVING FORM TWO STUDENTS' FAILURE TO SUBTRACT DIRECTED NUMBERS IN MATHEMATICS: AN ACTION RESEARCH STUDY

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Abstract

The purpose of this action research was to solve student's failure to subtract directed numbers in Mathematics at junior secondary (Form Two) level. The study was conducted at a secondary school in Gweru, Zimbabwe. The research had two cycles. Cycle 1 had four participants, while Cycle 2 had one participant whose problem had not been solved in Cycle 1. Cycle 1 had three sessions: structured interview, implementation of the cooperative strategy and evaluation of the effectiveness of the cooperative strategy. The cooperative strategy was used in conjunction with the number line and the 'having money-owing money' techniques. Cycle 2 had two sessions namely: implementation of the counters model and evaluation of the effectiveness of the strategy. Results from the implementation of the cooperative strategy indicated that logical sequencing of concepts, group work and good presentation skills reduced possible misconceptions. The major reflection was that although the participants had seemed to have the same problem, the use of one strategy may not be effective as was the case for participant D. The results from the interviews highlighted the importance of continuously checking with the learners to reflect on the teacher's practices. Results from the 'two-colour counters' showed the effectiveness in maintaining interest, motivation and concentration. The major limitation of the study was that the number line and the counters model were difficult to use, time consuming and laborious when large numbers were involved. The research has implications on the teaching and learning of directed numbers involving integers, fractions, decimals and algebraic expressions. A call for further research is also made.

Keywords: *Solving students' failure, Subtraction, Directed numbers, Form two, Action research*

INTRODUCTION

Directed numbers are important in the learning of many mathematics concepts. An understanding of directed numbers is necessary in manipulating equations, co-ordinate and graph plotting, factorization and change of subject of formulae, transformations and vectors and also in functions and inequalities. According to Ministry of Primary and Secondary Education (2015), Form 1 learners should be able to "operate with directed numbers" and "apply directed numbers to practical situations in life." (p. 13). However, we discovered during our teaching that many students enter Form 2 level with severe gaps in their concepts and skills in mathematics, especially in subtraction of directed numbers. Similar findings have been echoed (e.g., Bruno & Martinon, 1999; Makonye & Fakude, 2016; Sahat, Tengah & Prahmana, 2018). These are requisite concepts without which students cannot succeed in learning Mathematics and other related subjects. We recognized therefore that a firm basis should be put in place at Zimbabwe Junior Certificate (Forms 1-2) level if pupils are to fully understand the topics in Form 4 to Upper Six.

Action research methodology has been proposed and used by many researchers (e.g., Kemmis & McTaggart 1988, Segal 2009) as an effective problem-solving tool. Amini, Helmanto, and Hidayat (2020) conducted an action research study to determine students' understanding of operations with positive and negative integers. There results showed increased learning activities and increased learning outcomes by the students. We therefore decided to embark on an action research approach to solve our own problem.

This action research study is organised into three sections. Section 1 is the pre-intervention phase and includes the background to the study, identification of the problem, statement of the problem and research questions. The section concludes by giving the intended intervention strategy. Section 2 is the intervention phase and gives the plan of action, implementation of the plan and the results. Presentation of data, analysis and possible reflections of each instrument are also covered here. Section 3 is the post-intervention phase which looks at the overall findings and reflections.

METHODS

This study used classroom action research methods through two cycles. The data in this study were taken from junior high schools in Gweru, Zimbabwe. The first cycle involved four students with three sessions: structured interviews, cooperative strategies, and evaluation, While the second cycle involved one with two sessions, namely the student counter model and evaluation. Data collection techniques with interviews, observation, and tests.

RESULT AND DISCUSSION

SECTION 1: PRE-INTERVENTION PHASE

Background to the study

Although the concept of subtraction of numbers is introduced at early years of primary level mathematics, the answers could only take positive values and a minimum of zero (Cajori, 1991). During the development of Mathematics Syllabus Form 1-4 (2015-2022), it was assumed that the learner has completed primary education and has basic knowledge of primary mathematics syllabus concepts such as number and operations (Ministry of Primary and Secondary Education, 2015). Based on this, we were then convinced that by the time pupils enrol for Form two, they already have knowledge of subtraction of numbers acquired from the primary school mathematics syllabus (Curriculum Development Unit, 2006) as well as understanding of directed numbers as covered during Form 1. Within the primary level framework, the students should be able to subtract directed numbers between zero and ten thousand. One of the co-researchers of this study had this to say,

> *During my teaching of directed numbers to Form two West, I encountered a problem with four students who had serious difficulties in subtracting directed numbers. I realised that I needed to transform my practice, as I and the syllabus had been putting huge emphasis on wrongly assumed knowledge on the students' subtraction of directed numbers. The knowledge and skills I used in my teaching style were obviously to the detriment of the students. By placing much emphasis on the demonstration teaching method, I tended to neglect those students who did not have directed number skills and therefore, exposing them to a continuous experience of negativity and failure in mathematics and doing very little for their self-esteem and self-concept.*

This challenged us, therefore, to think of new ways in which we could seek for a suitable intervention strategy to solve students' failure to subtract directed numbers. The study was based on the premise that students' failure to subtract directed numbers was the key inhibiting factor in the students' learning capacities and potentialities. Thus, through a suitable intervention strategy, the problem could be eliminated leading to the students' acquisition of the requisite subtraction skills and successful learning of mathematics.

Purpose of Study

Thus, the purpose of this action research is to solve the problem of failure to subtract directed numbers faced by Form two students at a secondary school in Gweru, Zimbabwe.

Statement of the problem

The problem was that four students could not subtract directed numbers despite the fact that the subtraction concept had been taught in 4 consecutive lessons.

Research questions

The following research questions guided this action research.

1. Why are Form 2 students failing to subtract directed numbers in Mathematics?
2. What are the specific errors made by Form two students with regards to subtraction of directed numbers?
3. How can the 'having money- owing money' strategy be used to solve students' failure to subtract directed numbers?
4. How effective is the cooperative learning strategy in solving students' failure to subtract directed numbers?

Intended intervention strategy

The intended intervention strategy that was to be used in this action research to solve the problem of failure to subtract directed numbers is the cooperative learning strategy (as also advocated by Stoner, 2004; Cornelius-Ukpepi, Aglazor, & Odey, 2016) in conjunction with the use of number line and the 'having money-owing money' as teaching and learning techniques. The techniques will enable the use of concrete material in the teaching and learning of subtraction of directed numbers which makes this seemingly abstract concept interesting and real. Cooperative learning is a learner-centred instructional strategy in which students work together in small, heterogeneous groups to complete a problem, project, or other instructional goal, while teachers act as guides or facilitators. The cooperative learning strategy provides opportunities for productive struggle, in which students learn from their mistakes through explanations from their peers and teacher. The strategy also creates a positive dependency between group members as they work out problems on subtraction of directed numbers and can only finish successfully together. The strategy also provides supportive interaction between the group members as they attempt subtraction of directed number problems. The strategy also promotes both individual and group responsibility as they subtract directed numbers in groups. Furthermore, participants learn how to communicate in an appropriate way in small groups as they work out subtraction problems together.

The strategy utilises assessment of the group work and hence promotes collaboration between the group members. Since the group members want their group to attain good marks, they will ensure they promote peer learning, and put maximum effort to their task as a group. Thus, the use of the cooperative learning strategy and the techniques will provide the teacher with great potential to use their creativity to do further work on mathematics concepts as an alternative to merely relying on worksheets (Furner, Yahya & Duffy, 2005). The techniques utilize concrete objects used to help students 'understand abstract concepts in the domain of mathematics' (McNeil & Jarvin, 2007). The use of these tools in teaching will entail teacher's good purposeful planning and skills. The idea that children learn best through interacting with concrete objects has sparked much interest in the use of mathematics manipulatives such as the having money-owing money and number line, which are concrete objects that are designed specifically to help children learn mathematics (Ball, 1992; Amini, Helmanto & Hidayat, 2020).

The cooperative learning strategy was specifically chosen for the above merits and its potential to solve the problem of students' failure to subtract directed numbers. Thus, the strategy was expected to yield its intended purpose as had also been achieved by other researchers (e.g., Oni, 2018; Stoner, 2004) and advocated for by Cornelius-Ukpepi, Aglazor and Odey (2016).

SECTION 2: INTERVENTION PHASE

Plan of Action

This plan of action had two cycles. Cycle One had three sessions. The cooperative learning strategy was to be used to solve the four participants' (students') failure to subtract directed numbers. The cooperative learning strategy was to be used together with the number line and the having- money-owing money techniques which provided concrete objects (Ulrich, n.d., Wessman-Enzinger & Mooney, 2019) and facilitated the transition from concrete thinking to the abstract one in the conception of subtraction of directed numbers. This would facilitate the learning approach to start from the known to the unknown.

The participants were divided into two sets of pair-groups. Thus, there were four participants (coded as A, B, C and D) altogether. The pair-groups would allow the participants to discuss among themselves without my interference. This cooperative learning strategy promotes more free discussions of subtraction of directed numbers to take place between peers. Participants would practice in pairs the questions given to them. The teacher (first researcher) would only play a facilitative role in their learning. Mainly chalk and chalkboard were used to demonstrate and explain concepts in both cycles. After discussion, the participants would be given group work as well as individual exercises, which helped to check whether the set objective had been achieved or not. Data were collected through interview schedules, observation checklists, exercises and test. The sessions started by understanding the participants' problems through conducting a structured interview. This was then followed by the implementation of the plan which comprised of six activities and the last session ended by an evaluation exercise which intended to check on the effectiveness of the cooperative learning strategy. Continued failure by participant D, as was highlighted in the evaluative test after the implementation of the cooperative strategy, led to Cycle 2.

IMPLEMENTATION OF PLAN

Cycle 1

This cycle detailed the actual implementation of the planned three sessions and activities for the generation of data on subtraction of directed numbers. The sessions comprised of structured interview, implementation of strategy and effectiveness of strategy. These sessions were meant to answer the research questions in this study. Each session had a plan, implementation, results and observations and reflections. The four participants participated in all the three sessions.

Session 1: Structured interview

A structured interview with the four participants failing to subtract directed numbers was conducted. The purpose of the interview would also be explained before the interview, to ensure that the participants would be aware and comfortable in giving responses. The structured interview was to be conducted from 1300 hours in the participants' classroom. The participants' responses were recorded verbatim and their non-verbal communication was observed. The interview with each participant was planned to take at most 20 minutes.

Implementation of the interview plan

The interview with each of the 4 participants was conducted, one after the other. The purpose of the structured interview was to understand why the participants were failing to subtract directed numbers.

The interview session was based on the following questions:

1) Why are you failing to subtract directed numbers?
2) What is it that I do that makes you fail to subtract directed numbers?
3) What do you expect me to do when teaching subtraction of directed numbers?

Table 1: Participants' responses to structured interview, n=4

Interview question	**Responses**	**Participants**
1). Why are you failing to subtract directed numbers?	a) I cannot tell the sign associated with the directed number.	A, B, D
	b) I don't know when the sign entails subtraction operation and when it entails number sign.	A, B, C, D
	c) The concept of subtraction of directed numbers lacks real life examples	A, B, C, D

2). What is it that I do that makes you fail to subtract directed numbers?	d) Your demonstrations and explanations are not clear e) You give us inadequate time to practice f) You mix several concepts at a go g) Your demonstrations are too few h) You don't vary your teaching methods i) You don't give us real life examples	A, B, D A, B, C, D A, B, C, D A, C, D A, B, C, D A, B, C,
3). What do you expect me to do when teaching subtraction of directed numbers?	j) Give clear explanations when presenting subtraction concept k) Present on a single concept at a time l) Give us more practice time m) Vary your teaching methods n) Use real life examples	A, B, D B, C, D A, B, C, D A, B, C, D A, B, C

Results and observations

From the first question, responses indicate that participants A, B and D could not tell the sign associated with the directed number, which would then imply that the concept of subtraction of directed number was too complicated for them without this background knowledge. All the 4 participants highlighted that they did not understand when the sign entails subtraction operation and when it just entails number sign (Sahat, Tengah & Prahmana, 2018). This implied that the participants had not understood the identification of signs associated with a number and the rules of subtraction operation. All the participants indicated that they felt that their understanding of the concept of subtraction of directed numbers would have been enhanced with the use of real-life examples.

From Question 2, responses from participants A, B and D indicated that the demonstrations and explanations during the presentation stage of the lesson were not very clear. All the four participants indicated that the teacher was giving them inadequate practice time to work on tasks. The 4 participants also indicated that the teacher was mixing several concepts in one goal. This implied that the participants' failure to subtract directed numbers was attributable to poor presentation skills. Participants A, C and D highlighted that the teacher's demonstrations were too few such that they were given exercises before they grasped the concept. All participants indicated that the teaching method was too teacher-centred and had little participants' involvement. This made the mathematics lessons boring because the learners would always be listening to the all-knowing teacher. Participants A, B and C further indicated that the teacher never gave real life examples. This however, failed to support the well-known theory which states that effective learning takes place through learning from the known to the unknown.

In relation to Question 3, participants A, B and D suggested that the teacher needed to give clear explanations during the presentation stage of the lesson. Furthermore, participants B, C, and D highlighted that the teacher needed to present one concept at a time. This would enable the participants to grasp the concepts. All the four participants recommended that they should be given adequate time to practice given tasks. In addition, they all suggested that the teacher needed to vary teaching methods and incorporate concrete objects in the learning of subtraction of directed numbers so that they learn, as also highlighted by Sen, Tengah, Shahrill and Leong (2017) from concrete objects to abstract concepts.

Reflections

In our view, the interviews were successful because we gained more information than had been initially anticipated.

From the responses, we learnt that the teacher's teaching method was boring and was not motivating to the participants, thus the teaching strategies had to be changed. We also thought of incorporating a variety of media and concrete examples as a way of enhancing teaching. The interviews also reflected to us that the teacher needed to logically sequence the development of subtraction of directed number concept. However, a notable weakness in the interview was that the teacher conducted the interview in which participants had to divulge her areas of weakness hence participants at first found it difficult to tell her what she was not doing well.

From the results of the interview in Session 1, we noted the importance of incorporating the cooperative learning strategy as a variation to the explicit strategy that the teacher had been using. This then led to Session 2, which was the implementation session.

Session 2: Implementation of cooperative learning strategy

Session 2 was meant to address the problems resulting from the interviews in Session 1. The session consisted of six activities which were meant to gradually develop the concept of subtraction of directed numbers in an endeavor to solve participants' initial failure to subtract directed numbers. Four participants participated in the six activities of the implementation of the intervention strategy. As was suggested in interviews, the participants had to be given more practice time, use concrete objects, and use real life examples to enhance the grasping of the concept. The cooperative learning strategy promotes the use of group work/ peer learning as a better learning strategy compared to individual and competitive learning (Oni, 2018; Cornelius-Ukpepi, Aglazor & Odey, 2016; Stoner, 2004). The participants were divided into two heterogeneous and permanent pair groups throughout the implementation of the strategy. The use of permanent groups meant that the peers would be jointly assessed. It also helped in developing the spirit of "ours" and not "my", which promoted joint effort in achieving the objectives. Each pair recorded on a worksheet the work attempted in each activity. The teacher/researcher also used observation checklist to complement the responses from worksheets. The six activities are detailed below.

Activity 1: Identification of positive and negative numbers.

Our objective in giving this activity was to assist the participants to be able to identify positive and negative numbers, as a basis to the understanding of subtraction of directed numbers. We defined directed numbers as all the numbers written with a positive or negative sign (i.e., with a direction). Zero is in the neutral position and numbers to the right of zero or numbers greater than zero (0) are positive, while numbers to the left of zero or numbers less than zero are negative. The teacher used the number line to demonstrate and explain on the chalkboard, relating the distance and direction of the number from zero as a way of identifying positive and negative numbers.

The participants were given guided practice questions which they worked in pairs. For example: Identify these (a) -10 (b) +48 and the expected answers would be negative 10 and positive 48, respectively. Five such questions were given. Further, participants engaged in individual work on identifying positive and negative numbers. Participants B and C initially failed to identify +123 as a positive number but got it correctly at the second trial.

Results and observations

All the participants did well. The results from the pair work indicated that the participants could now identify the signs associated with the number, regardless of the number being single, double or triple digit.

After having observed that the participants could identify the signs associated with numbers from pair work responses, the teacher then went on to further demonstrate that the "+" sign of a positive number may be omitted. For example, "+5" could be written as or is the same as 5, but negative numbers always carry their sign with them, otherwise leaving the sign will imply that the number is positive.

Reflections

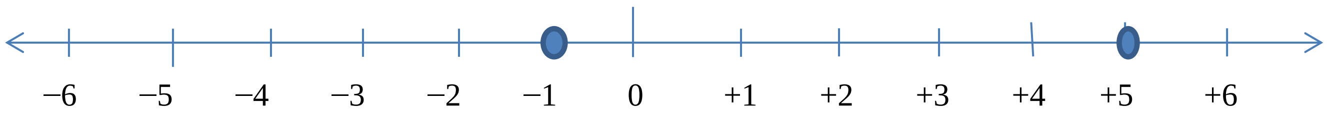
The results from the pair work reflected to us that participants could work better when they assist each other. We attributed the good results as the benefits of cooperative learning strategy. As the participants were observed working in their respective pairs, we noticed that the strategy encouraged peer learning within the pairs.

From both the individual exercise and pair work results, we concluded that the participants could now identify the sign associated with a number, when the direction is directly indicated. The results for Question 5 of the individual exercise indicated that participants B and

C had initially not grasped the concept of eliminating the sign for positive numbers. We could attribute this failure to the fact that only one example had been given, which could have been missed by the two participants.

Activity 2: Drawing and representing directed numbers on a number line

The purpose of this activity was to assist the participants to draw a number line and represent directed numbers on it. The teacher used a number line drawn on a manila sheet as well as ones drawn on the chalkboard. The teacher demonstrated how to circle the positions of (i) -1 and (ii) 5 on the number line as follows:



Pupils were then asked, in pairs to do five similar problems.

Results and observations

Participants B and D did not manage to draw the number line correctly. They skipped -9 on labelling the number line. Such a mistake explains the possibility of the participants getting wrong answers when subtracting directed numbers. The mistake was noticed and corrected when the teacher was observing and marking the pairs' work sheets. It was observed that the cooperative learning strategy promoted group responsibility. Both participants B and D later managed to correctly circle the position of the numbers on the number line.

Reflections

In terms of the pairs' ability to circle the number on the number line, we felt that this was enabled by the skills developed in Activity 1. Circling the position was easier considering that in Activity 1 they had gained the knowledge that positive numbers were greater than zero or to the right of zero, while negative numbers were numbers less than zero or numbers to the left of zero. More independent practice questions with two-digit numbers were later given.

Results and observations of practice questions

The results indicated that all the participants were able to draw and clearly label the number line stretching to 2-digit figures with no challenges. On the 'circling of position' exercise, participant D failed to circle -12, because the participant had changed the question to 12 instead of -12 as given. When asked to circle the position of zero on the number line, the same participant forgot that 0 was neutral, and labeled it on the number line as + 0. In this case the participant related to the fact that any number without an indicated sign meant that it was positive, thus violating the neutrality rule and not fully grasping the concept of signed quantities (Ulrich, n.d.).

Reflections

The results reflected that the participants had mastered the concepts in the activities. The teacher needed to be careful in ensuring that the minor mistakes made by the participants were contained before they could develop into bigger challenges. The cooperative learning strategy, to this end, had been very useful in enhancing the mastering of the concepts covered so far; concepts which are part of the background to the understanding of subtraction of directed numbers.

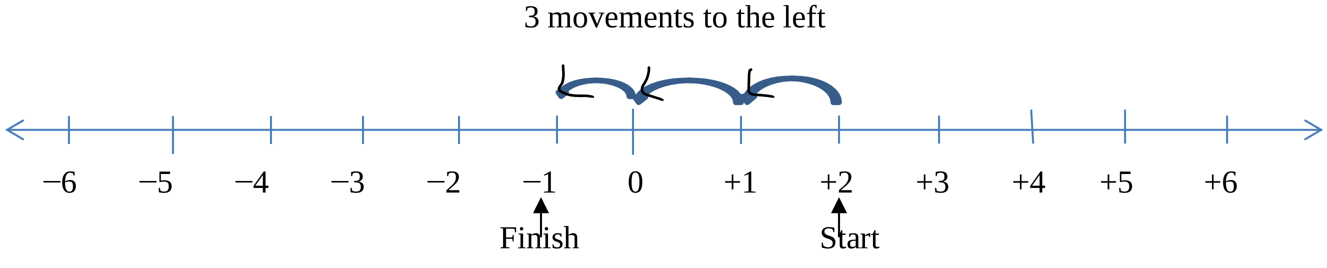
Activity 3: Subtraction of positive numbers using the number line.

The purpose of the activity was to assist participants to subtract positive numbers using the number line. This was only possible provided the teacher first explained the rules that govern the subtraction of directed numbers.

Rule 1: - (+) = - this entails subtracting a positive number. The negative sign is the subtraction operation sign and the (+) implies a positive number.

Rule 2: - (-) = + this implies subtraction of a negative number. The first negative sign represents the subtraction operation, while the second negative represents a negative number.

Rule 2 would be considered in Activity 4. For Rule 1, subtraction of a positive number results in movement in the negative direction. Thus, when a negative is followed by a positive, the resultant effect on the operation is negative. It is important to note that subtraction problems begin by giving the negative sign of the operation, before giving the number to be subtracted. For instance, given 2-3, one needs to remember that a number such as 2 that has no sign before it means it is a positive number. The negative implies subtraction sign, and 3 is a positive number too. In this case, one goes to the +2 on the number line, and since subtraction means moving to the left, one then moves 3 steps in the negative direction. This can be illustrated on a number line as follows: $2-3=-1$



After demonstrating another example such as $-1-3=-4$, the teacher then gave a similar 5-question exercise to be done in pairs.

Results and observations

Both groups managed to correctly draw and label the number line for use in subtracting positive directed numbers. In the first three questions, both groups managed to perform the subtraction operation using the number line. Pair 1 (A and C) moved in the opposite direction and obtained +9 instead of -9 for the problem 0-9. Pair 2 (B and D) also obtained +7 for 8-15 instead of -7. Maybe they thought that since the teacher said the concept relates to subtraction of positive number, then the answer would be positive. When this error was observed, the teacher further explained why correct and consistent use of the number line was still important at that stage.

Reflections

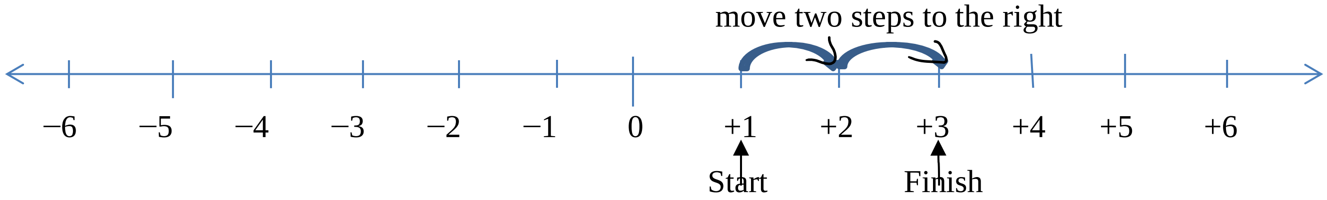
The use of the number line made it easier to achieve the set activity objective. The number line was an effective media on teaching the concept. Similar successful results using the number line had been observed in previous studies (Bruno & Martinon 1999; Amini, Helmanto & Hidayat, 2020; Sari & Jaguthsing, 2021). The use of cooperative learning strategy also proved helpful to the participants because the teacher was monitoring their practice and noticed that they would find time to explain the concept to each other, thus promoting peer learning.

Activity 4: Subtraction of negative numbers using the number line.

The objective in this activity was to help the participants subtract negative numbers using the number line. The teacher began by explaining Rule 2 which stipulated that -(-) = + thus implying subtraction of a negative number. The first negative sign represents the subtraction operation, while the second negative sign represents a negative number. It is important to note that subtracting a negative number is equivalent to adding a positive number. Therefore, to subtract a negative number, one moves in the opposite way of a negative direction, that is to the right. Generally, two consecutive negative signs give a positive sign, that is (-) (-) = +.

For example, to solve $1-(-2)$ by applying Rule 2 would result in 1+2. The number line could then be used to find the answer as follows:

Example: 1- (-2) = +3



The teacher also demonstrated and explained the following example: $(-1)-(-5)$.

In this case it is important not to confuse that the first negative, to the left of 1, imply negative number 1 (-1). By resolving the two consecutive negatives, one gets +. The question would then become -1 + 5. Then using the number line as before would result in -1- (-5) = +4. Participant C was then asked to demonstrate the example 0 − (−4) on the chalkboard with the assistance of fellow participants. The participant successfully solved the problem. The teacher gave the participants some more practice questions which they worked out in their pairs.

Table 2: Responses to pair-practice using number line to subtract negative directed numbers

Question	Response		Errors
	Pair 1 (A and C)	Pair 2 (B and D)	
1. (+2) − (−3)	✓	✓	No
2. (−6) − (−4)	✓	✓	No
3. (−3) – 0	✓	☒	Pair 2: obtained +3 instead of -3
4. 13 – (-5)	✓	✓	No
5. -13- (-5)	☒	✓	Pair 1: obtained -18 instead of -8

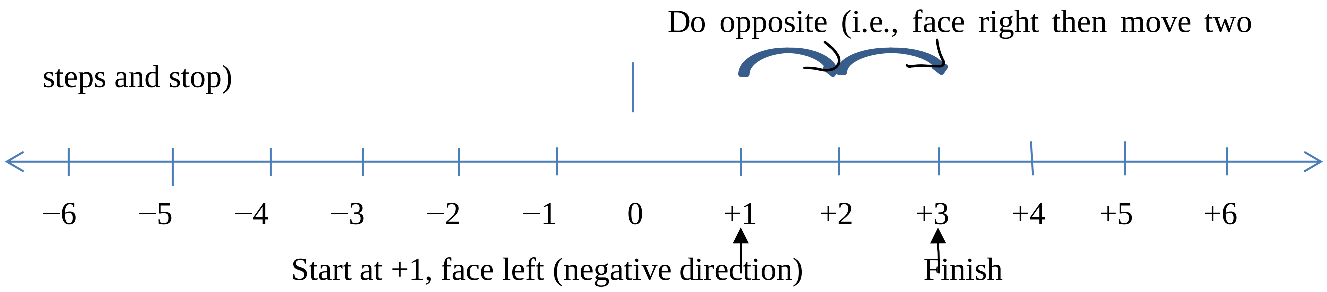
Key: ✓- able to ☒- unable to

Results and observations

The set objective was partially achieved. Pair 1 managed to perform the subtraction of negative numbers well except on question 5 where their answer reflected that they thought the question was completely the opposite of question 4. We noted that perhaps they did not follow the correct steps in subtracting negative directed numbers. Maybe they also did not make effective use of the number line as they had been advised. Pair 2 managed to do well in the first and last two questions and failed the third question. We observed that they encountered a challenge probably because they failed to represent subtraction of zero on the number line. They ended up applying the involved negative signs, as if there were two consecutive signs, thus, obtaining positive 3. When the teacher also observed the errors, she requested the other pair to demonstrate on why and how they had resolved the same problem in their pair. She emphasized the need to ensure that the negative signs are one after another (consecutive) for them to be positive and the need to follow the procedures and avoid making assumptions.

Reflections

We noted that the teacher could alternatively demonstrate the problem 1- (-2) as follows: Start at +1. Since we are subtracting, face the negative direction (i.e., left), Then because of the second negative sign, do the opposite (i.e., instead of left now face right) and move two steps, then stop. The answer is + 3.



The number line proved helpful in solving subtraction of negative numbers. We noted that the participants made errors maybe by not following the rules and procedures. However, the use of cooperative learning proved very helpful in developing teamwork and eliminating competition and rather promoted the spirit of group success. It also reflected to us that the participants had grasped the concept of subtraction of negative directed numbers. We reflected that the use of cooperative learning could indicate that some vocal group members might convince others and make errors, especially if group work was not divided to give each member responsibility.

The practice questions were diversified in terms of coverage. This enabled the participants to familiarize with each case and understand how to solve the involved subtraction in each case. The questions also included 2-digit problems as a way to expand the conceptual understanding by the participants. The participants were given the following independent practice questions: Use the number line to solve (a) +8- (-5), (b) – 7 – (-6), (c) 0- (-13), (d) -17 – (-3) and (e) 14 – (-5).

Results and observations

Participant C had mastered the concept of subtraction using the number line. Participants A, B and D had problems in question (c). We observed that they had similar misconceptions where maybe they thought it was just the same as moving 13 steps from zero, like in number line construction, to obtain -13. Participant D, in addition failed parts (b) and (e) which indicated that probably he had not grasped well the concept of subtraction of negative numbers.

Reflections

Although the use of the number line had proved helpful in solving subtraction problems, the teacher needed to clarify the difference between number line construction technique (putting signs) and subtraction operation techniques (Sahat, Tengah & Prahmana, 2018) and the need not to confuse the two. Regardless of the error, the use of the number line proved to have assisted participants A, B and C to master subtraction of negative directed numbers. However, the errors made by participant D highlighted to us that he still had challenges with the concept.

Activity 5: Combining subtraction of both positive and negative numbers using number line.

The purpose of this activity was to help the participants to use the number line to subtract combined problems on positive and negative directed numbers. As a way of checking whether the participants still recalled the skills, the teacher began the activity by asking the participants to use the number line to work out in pairs the questions such as $(-12) - (+3)$ and $(-7) - (-4)$.

Both pairs managed to use the number line to work out the above questions to obtain -15 and -3 respectively. The teacher only gave a few practice questions in this case because the concepts had been previously covered in Activities 3 and 4 and participants had worked in pairs in both cases.

Similar questions were given to work out as individual homework. The questions were $(+3)-(-3)$, $(-3)-(+3)$, $(-17)-(-2)$, $(+2)-(+16)$, and $10-(-10)$. This was meant to establish whether the participants had fully mastered the subtraction of directed number skills using the number line or not.

Results and observations

Participants A and D were unable to work out $(-3)-(+3)$. Participants A and D got 0 and +6 respectively, instead of -6. They failed to apply Rule 1 of consecutive opposite signs, which implied that – (+) = -. This meant the question would be -3-3 to get -6 instead. Participant A got -3 +3 to obtain 0 and participant D repeated the error of assumption and probably thought that $(+3)-(-3)$ and $(-3)-(+3)$ were the same and jumped into conclusion that the answer was 6.

Reflections

The objective of the set activity was partly achieved. Generally, all the participants could use the number line to subtract directed numbers, except participant D. The participants could now work in groups well and helped each other to subtract directed numbers. However, the use of the number line limited the subtraction to smaller figures which could be represented on the given number line. Subtraction involving bigger figures seemed to render the number line technique useless. The over-reliance on the number line may entail problems especially with large numbers which makes the drawing and labeling time consuming, laborious and monotonous. This then led the researchers to think of another technique which could address the weaknesses of the number line.

Activity 6: Subtraction of directed numbers using 'having money-owing money' technique.

The objective in this activity was to assist the participants to subtract directed numbers involving both small and big figures through the 'having money-owing money technique.' With this technique, the teacher made use of abstract money since real or concrete money was not available and because all participants were familiar with money in dollars ($) and also familiar with monetary gains and losses as in bookkeeping (Wessman-Enzinger & Mooney, 2019). Aris, Putri, and Susanti (2017) found out that the use of multimedia such as money helped students to understand addition and subtraction of integers.

The teacher used money which is easier when building on abstract understanding. Firstly, the use of having money-owing money technique in performing the subtraction operation of directed numbers was explained. Using this technique, a positive number represents the money that one has, whereas a negative number represents money that one owes others. Given numbers were interpreted as dollars ($). Using this technique, the rule is that if one has money and owing others at the same time, one has to pay what s/he owes others first even if it does to settle the debt.

For example, 30 – 50 means that I have $30 (+30) and I owe others $50 (-50). This means that by the rules of this technique, I have to use the $30 to pay what I owe others first. I would be able to pay $30 of what I owed someone and I will be left owing $20 which is written in directed number form as -20.

The teacher demonstrated and explained the following second example: Given (+18) – (+7), the first thing was to apply the rules of consecutive signs and get (+18) – 7. This meant that a person had $18 but at the same time owed $7. Thus, one had to pay what was owed to others and would be left with $11, which is the same as +11, in directed number notation.

The following example was also demonstrated: -100 – (-25). The first step was to apply the rule of consecutive signs to get: -100 +25. This now meant, one owes someone $100, but at the same time one has $25. So, one has to pay back the $25 and ends up owing $75 which is the same as -75 in directed number notation.

The participants were further given similar guided practice questions, which they answered through dramatization in their pairs after resolving the consecutive rules. For instance, in the first question (+40) - (+50) and after resolving the consecutive signs, the pairs got (+40)-50. One participant in a pair had $40, but at the same time owed the second participant $50. So, the first participant paid the second participant the $40, and remained owing $10 which is equal to -10 in directed numbers. All the questions were attempted this way and all the participants did well.

Results and observations

The objective of the activity was achieved. The participants enjoyed the dramatization method of learning. The participants indicated that the having money-owing money technique had the strength of providing real daily life experiences which made it is easier to apply to the concept of subtraction of directed numbers. We observed that in Question 2 for instance, the participants managed to apply the concept that -35-35 was as good as a case in which one owing $35 borrowed some more $35 and ended up owing $70 which is the same as -70. We felt that the participants had grasped the concept of directed numbers well. After the dramatization exercise the participants were given more independent practice questions.

Results and observations on independent practice questions

All the other participants except participant D were able to subtract directed numbers using the technique of having money-owing money (practice questions). Participant D managed to correctly answer three of the six questions. We observed that participant D had probably not thoroughly grasped the money concept and some of the errors could have been emanating from poor subtraction skills or abilities.

Reflections

The technique of using money was easier to conceptualize because participants already understood money. The technique also was flexible enough to engage learners with difficulties on subtraction of directed numbers during the activity and thus, it could be manipulated to become fully learner-centred and addressed the problems of the participants. The technique also facilitated subtraction of larger numbers with little difficulty. The participants argued that the technique was easier to remember because of its practicality, than other techniques which may promote rote learning as one needs to cram that moving to the left means negative, and to the right means positive. The use of this technique within the strategy of cooperative learning allowed the participants to 'move away', on subtraction of directed numbers, from the concrete operation to the abstract one as proposed by Jean Piaget. The conceptualisation stage allowed the setting of higher order questions involving directed numbers.

SESSION 3: EFFECTIVENESS OF COOPERATIVE LEARNING STRATEGY

This session was meant to test the effectiveness of the strategy. The teacher was not interested in the test scores of the participants, but instead, the participants' ability to perform the subtraction operation on directed numbers. The teacher utilized a review assessment task to evaluate the effectiveness of the strategy. The participants answered the evaluative test independently. All the concepts involved in the previous activities (1-6) were covered in the test. The test had 10 questions which were to be answered in 45 minutes.

Results and observations

The strategy of cooperative learning was partly effective in solving participants' failure to subtract directed numbers. Three of the four participants could now subtract directed numbers with little difficulty. Participants A, B and C proved to have fully mastered the concept of subtraction of directed numbers through the use of cooperative learning strategy. Participant D seemed not to have mastered the concept, particularly with the use of the number line. Although in group work, the pair seemed to do well, it could then be argued that the performance of participant D could be attributed to participant B's mastery. In this case, it can then be concluded that the use of cooperative learning strategy failed to reflect participant D's failure, as it remained covered by B's contribution to pair work.

Reflections

The errors initially made by the other participants were remedied by the use of the cooperative learning strategy. The implementation of the strategy was logically sequenced which facilitated concept development. In implementing the strategy, the teacher also incorporated media which made it easier to explain the abstract concepts. The strategy utilized varied activity approaches including dramatization. The use of varied techniques enabled the teacher to keep the participants motivated and engaged. The strategy was more learner-centred and kept engaging the learners in activities. This gave participants ownership of their own learning and they were responsible of their success. Apart from contributing to participants' mastery of the subtraction concept, the strategy also promoted other social skills such as team-work and good communication skills. However, the use of this strategy may fail to achieve the intended results when group members are not comfortable working together. In this case the group members may work in competition and not collaboratively, which we suspect was the case for pair 2 and minimal peer learning would thus take place. This could explain the case for participant D's failure.

Failure by participant D to compute most of the subtraction problems in Cycle 2 indicated to us that the cooperative strategy had failed to successfully solve the participant's failure to subtract directed numbers. The researchers felt that the teacher could not leave the participant with the problem unsolved. They then considered proceeding to Cycle2 with participant D using the Counters Model as the new intervention strategy.

Cycle 2: Plan of action

The plan of action for Cycle 2 had two sessions. The first session was the implementation of the counters model as the intervention strategy. In the second session the researchers evaluated the effectiveness of the counters strategy in solving the participant's failure to solve directed numbers. The two-colour counters strategy was to be used. The plan of action for the teacher is detailed in the table below.

Table 3: Cycle 2- Plan of action

CYCLE 2			
Sub Research question	**Session**	**Activities**	**Data generating tools**
1. How can I use the counters model/ strategy to solve students' failure to subtract directed numbers?	**Session 1: Implementing the intervention strategy**	Activity 1: Using the counters model to subtract positive directed numbers.	Exercises, observation check lists
		Activity 2: Using the counters model to subtract negative numbers	Exercises, observation check lists
		Activity 3: Using the counters model to subtract both positive and negative numbers.	Exercises, observation check lists
2. How effective is the counters strategy in solving the Form 2 pupils' subtraction problems?	**Session 2: Effectiveness of strategy**	Review assessment task on subtraction of directed numbers.	Test

The cycle had two sessions: implementation of intervention strategy and effectiveness of strategy. The implementation of intervention strategy had three activities.

Session 1: Implementation of intervention strategy

This details the implementation of the counters model as an alternative to the cooperative learning strategy that was used in the previous cycle. The researchers chose the counters model because it utilizes counters of different colours which help to motivate and maintain the interest of the participants. The session began with a plan as detailed below.

Plan

The teacher intended to implement the counters model with one participant who had failed to subtract directed numbers. The teacher planned to use an exercise and observation checklist to record the responses to the activities done by the participant. It was intended to use a variety of instructional methods which are more participant-centred. When implementing the counters model, the teacher intended to have the first activity concerned with subtraction of positive directed numbers and the second activity would be on subtraction of negative directed numbers. The last activity in this session would be on subtraction of both positive and negative directed numbers.

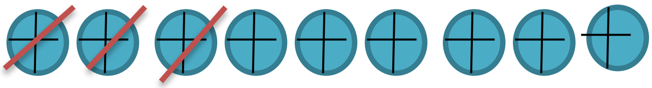
ACTIVITY 1: Using the counters model to subtract positive directed numbers.

The purpose of this activity was to assist the participant to subtract positive directed numbers using two-colour counters. The teacher assumed that the participant knew positive and negative numbers. She began the activity by explaining the two-colour counters model (Sahat, Tengah & Prahmana, 2018; Sen, Tengah, Shahrill & Leong, 2017).

When using two-colour counters to represent directed numbers, the blue counters represent positive and red counters represent negative numbers. It is important to note that there is a difference between negative and minus. Positive and negative signify the sign of the directed number. Plus or minus signify whether one has to add (plus) or subtract (minus). In this model, one should think of subtraction as take away, which means actual removal of things. 'Take away' also means one **does not** present both numbers of the subtraction question, only the first number.

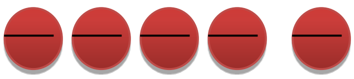
For example, to work out 9 - (+3) using counters may be demonstrated as follows:

Both 9 and 3 are positive. Remember when performing subtraction operation, one represents only the first number. From this example only 9 counters are presented and they represent 9 positive numbers. The question requires that one takes away 3 positive counters. Since the 9 counters are positive, then one simply takes away the 3 counters, indicated by cancelled counters to remain with 6 positive counters (= +6).

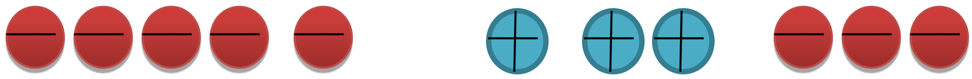


To subtract the (+3) from -5 using the counters model, one needs to also familiarize with the technique of obtaining zeros (0) as a neutral intervention as demonstrated by (Sahat, Tengah & Prahmana, 2018) where available counters cannot satisfy the required subtraction operation. With this technique, the addition of a positive counter and a negative counter at the same time, gives a neutral (zero) result, but would allow one to perform the required subtraction operation. This technique is also applied where the available counters are not adequate to perform the required subtraction operation.

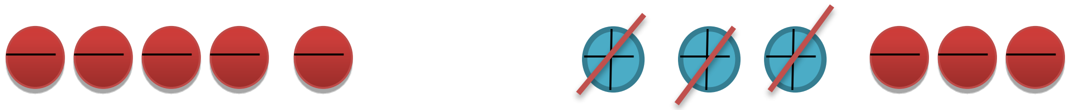
From the example -5 – (+3), one starts with 5 negative counters as follows:



The question requires the teacher to subtract 3 positive counters, which she does not have. In this case she brings the 3 sets of neutral counters, that is, the zero counters. She adds an equivalent number of both positive and negative counters as a way of bringing the required positive counters to picture and enables the subtraction operation to be performed. The number of the required counters should always be matched with the number of the neutralizing counters, such that the overall effect of adding the counters will be zero. This is done as follows:

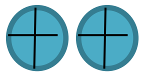


From the above illustration, the teacher now has the 3 positive counters to take away and this will be done by cancelling the counters as follows:

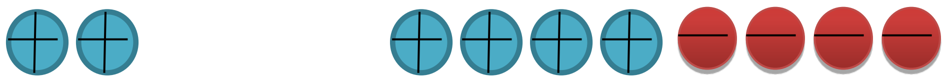


In this case the remaining 8 negative counters is the answer, which is -8.

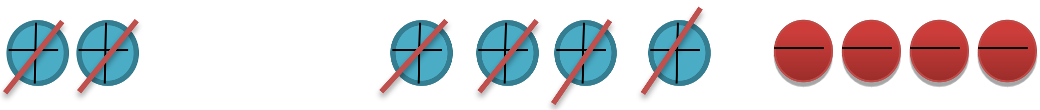
The teacher also demonstrated and explained the use of the counters in the problem of (+2) – (+6). In this case she started with 2 positive counters as follows:



But she was required to subtract 6 positive counters and had only 2 positive counters. This means she should add 4 zero sets as follows:

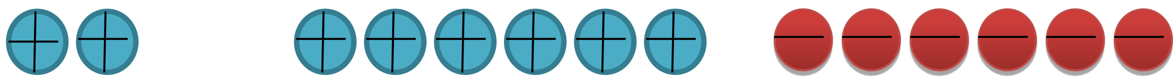


Now she could subtract the 6 positive ones as follows:

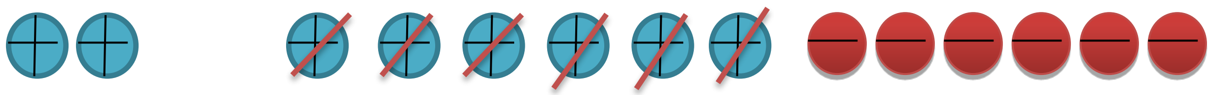


Her answer is the remaining counters with their respective sign, that is, negative 4 = -4

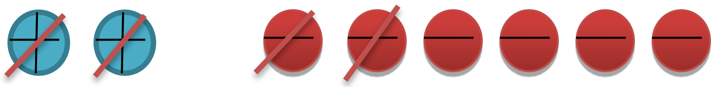
Alternatively, given (+2) – (+6), one could then add the 6 zero sets as follows:



From the above, one can now subtract the 6 positive counters as follows:



One remains with 2 positive counters and 6 negative counters. Recalling from the zero set, the combination of a positive and negative counter gives a zero result. So, the 2 positive counters will cancel out with two negative counters as follows:



Thus, the final answer is -4.

The teacher then gave the participant guided-practice questions and the results were as follows: For 8 – 11, the participant failed to utilise the concept of adding zero set while for 0-5 the participant was confused by starting with no counters.

Results and observations

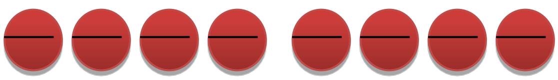
It was observed that the participant failed in the parts stated above because he could not apply the zero-set effect to these problems to enable the subtraction to be performed. We noted that the teacher needed to give more practice questions incorporating the 'zero-set.'

Reflections

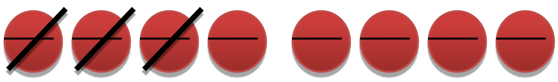
At first, we thought the counters model would be too difficult for the participant to comprehend. We were surprised to notice that the participant could work with little difficult than expected. The model incorporates a diversified range of skills, which promotes development of critical and logical thinking in subtraction of directed numbers. The counters model had also been used, with successful results, by (Sahat, Tengah & Prahmana, 2018). Our participant showed in this study that he had grasped the concept of subtraction using counters but needed further practice to ensure that the errors would be eliminated.

ACTIVITY 2: Using the counters model to subtract negative directed numbers.

The objective in this activity was to assist the participant to subtract negative directed numbers. The teacher began the activity by demonstrating how she could use the counters to solve -8 – (-3). She started with 8 negative counters as follows:

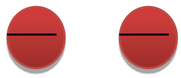


From these counters, one should take away 3 negative counters. The teacher showed this by cancelling the 3 negative counters as shown below:



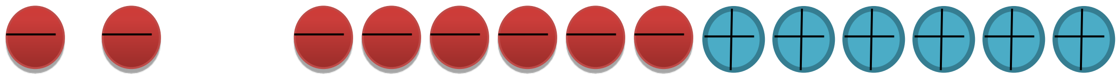
The answer was represented by the remaining non-cancelled 5 negative counters, = - 5.

The participant also demonstrated using counters (-2) – (-6) on a worksheet:

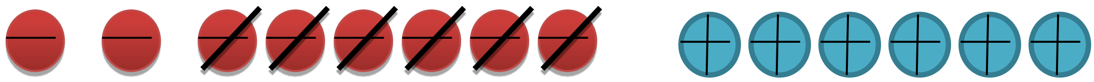
The participant started with 2 negative counters as follows: 

From which 6 negative counters should be subtracted.

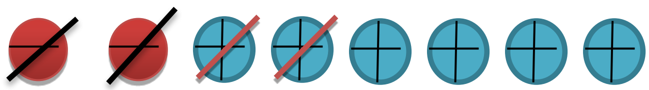
The participant then applied 6 zero sets to obtain the following:



This then allowed the participant to subtract the 6 negative counters as follows:



The participant then remained with 2 negative counters and 6 positive counters. The participant went on to apply the zero-set effect, so that the 2 negative counters cancelled out with 2 positive counters as follows:



Thus, the answer was 4 as indicated by the 4 positive counters remaining.

The teacher requested the participant to demonstrate on the chalk board what would happen in case of 6 – (-1) of which the participant did well.

From this demonstration, the teacher noticed that the participant had grasped the concept of counters and she had to move away from demonstrating the examples herself but shift to a participant-centred approach. In this case the participant was doing much of the demonstrations and practice which would maximise the participant’s interest and motivation. This allowed the teacher more time to observe the participant’s activities and guide him immediately when a misconception or error occurred.

After more participant-centered demonstrations which were well done, the teacher then gave the participant some five independent practice questions. These were all well done as well.

Results and observations

The results indicate that the participant could now solve subtraction of directed numbers using the counters. We observed that the participant enjoyed the counters strategy in solving subtraction of directed numbers. His participation had significantly increased. The participant enjoyed working with colourful counters. The participant indicated that using different coloured objects was easier to visualize what would be happening, unlike with similar objects.

Reflections

The above results indicated to us that the teacher had underestimated the participant's ability to comprehend the counters model. Initially, she thought this would be very difficult to the participant, such that she incorporated it with skepticism.

ACTIVITY 3: Solving subtraction problems involving both positive and negative directed numbers.

The purpose of this activity was to assist the participant to solve subtraction problems involving both positive and negative directed numbers. Results from the previous activity highlighted that the participant had mastered the concept of using counters to subtract directed numbers, such that in this activity the teacher began by giving the following questions as guided practice.

Table 1. Participant's results on the use of counters to solve guided practice questions

Question	**Expected answer**	**Same result**
1. Use the counters to indicate which subtraction statements have the same results.		
a) (+9) – (+5)	+4	☒
b) (-4) – (-9)	+5	☑
c) (+7)- (+2)	+5	☑
d) (+2) – (-3)	+5	☑
e) (-4) – (+1)	-5	☒
f) (-3) – (-2)	-1	☒

Key: ☑- same result ☒- not same result

Results and observations:

The participant managed to use the counters model to obtain the above responses. The participant then indicated that only b), c) and d) had the same answer of +5. The participant managed to use counters to obtain the answers. He then compared the answers and indicated similar ones. The process was only possible with the correct use of counters otherwise failure would result in wrong answers and wrong decisions.

Reflections

More questions which ranged from single digit to 2-digit numbers were given and this allowed the teacher to check if the participant could apply the concept to higher order questions. Since in these sessions, only one participant was involved, the easy grasp of the concept can be attributed to the benefits of one-on-one instruction method. Working with one participant enabled the teacher to address the areas of misconceptions as they arose.

SESSION 2: EFFECTIVENESS OF THE STRATEGY

This session was an evaluative one. The purpose was to assess the effectiveness of the counters model to solve the participant's failure to subtract directed numbers. The teacher used a review assessment test with 10 questions, covering concepts in Activities 1-3.

Results and observations

The participant managed to solve all the questions on subtraction of directed numbers using the counters model as the teacher expected. The results indicated that the strategy was effective in solving the participant's failure to subtract directed numbers. The participant's ability to solve both single and two-digit problems also indicated mastery of the concept.

Reflections

The strategy was effective in solving the participant's failure to solve subtraction of directed numbers. The use of two-colour counters helped to maintain the participant's interest and motivation because the participant generally liked colourful objects. The success of the strategy can also be attributed to the fact that the participant already had an appreciation of the background information, which allowed the teacher to start at the implementation stage of the counters. The 'zero-sets' was another mathematically important concept, whose use enables understanding in other mathematical concepts such as equations and algebra (Sahat, Tengah & Prahmana, 2018; Sen, Tengah, Shahrill & Leong, 2017). However, the use of the counters was limited to smaller values that could only be represented by counters without the work being clumsy and tedious. This was difficult for the teacher to explain to the participant, and hence she deliberately left such questions. Also, the use of counters entailed concrete objects, which could possibly make it difficult to shift to abstract and conceptual understating of the concept. The strategy was more effective than we had initially thought it would be, considering that the participant was now working without peers.

Summary

The section began by detailing the plan of action which comprise two cycles. Cycle 1 had three sessions namely: structured interview, implementation of cooperative strategy and evaluation of the effectiveness of the strategy. The sessions addressed the four research questions in this action research. Cycle 1 had four participants and three of these had their problem solved by the cooperative learning strategy. Cycle 2 had one participant. The cycle had two sessions namely: implementation of counters model and evaluation of the effectiveness of the strategy. The strategy managed to solve the participant's failure to subtract directed numbers, beyond the teacher's expectation. Since all the participants' problems had been solved, Session Two of Cycle 2 brought the research to the end of Section 2. In the next section, overall findings and reflections will be highlighted and discussed.

SECTION 3: POST-INTERVENTION PHASE

Introduction

This section focuses on presenting the overall findings and reflections. Opportunities and limitations which were experienced are also discussed. Summary, concluding thoughts and recommendations sum up the section.

Overall findings and observations

A summary of the students' errors with regards to subtraction of directed numbers were noted and some are listed as follows:

a) Incorrectly identifying positive and negative numbers on the number line
b) Not observing and not correctly using the subtraction rules, -(+) = -, +(-) = -, and -(-) = +
c) Failure to distinguish – as an operation and – as a sign of a given number
d) Failure to correctly apply the 'zero-set effect' when using the two-colour counters
e) Failure to do the subtraction itself or the arithmetical computations, either mentally or by use of media.

One major finding of this research was the need for the researchers to be flexible enough to be able to use action research to solve some of the classroom problems as mentioned earlier. This entails us becoming 'teacher-researchers.' In this case one would enhance one's teaching practices. The research also highlighted to us that when using the explicit strategy in the teaching practice, the teacher would be probably assuming that the learners were empty vessels, which proved not true from the sessions conducted. This also highlighted the need to move away from teacher-centred strategies to learner-centred ones as was reflected in the results. We also observed the importance of first establishing the learners' abilities. We observed that the teacher should not just rely on assumed knowledge of the syllabus but should be creative and innovative. We also noted that the syllabus assumed an all-abilities class which is not what was actually experienced in practice. In addition, we needed to logically sequence concept development starting with what the learners know (known to the unknown approach) and the use of real-life examples and concrete objects proved helpful. The use of media and colourful objects had positive learning effects. Furthermore, we observed that the learners were bored and not motivated to sit and listen to the teacher always, yet we thought the teacher's thorough explanations would lead to learning. We observed that peer learning was more effective, instead. In the research we also found out that the teacher was not giving the learners adequate time to practice, which exacerbated the problems encountered in Mathematics. We noted that the use of the 'counters strategy' was enhanced by the use of 'one-on-one' to promote learning to take place.

Reflections

One of the major observations was that the teacher constantly checked with her learners to reflect on her practices. The teacher herself had this to say:

> *I was surprised to note that I had poor presentation skills, as I used to mix several concepts at one goal. I then attributed part of the learners' failure to myself. This was very important as I could tell this myself.*

The instruments used to collect information were effective because all the research questions were answered. For instance, the interview allowed the researchers to understand the reasons behind the failure, which would then need to be addressed. The placement of the interview in the first session allowed the logical sequencing of the subsequent sessions. The use of a structured interview enabled key issues to be considered and to ensure consistence of questions asked and responses obtained. The interview highlighted issues that maybe the teacher had never thought of, for example, the use of varied teaching approaches as well as the need to give adequate practice time. The participants' failures in one session were addressed in the subsequent sessions. However, the use of structured interview could be argued to limit the respondents to the covered questions only. This means that any responses outside the scope of the questions could be deemed useless.

The research also reflected the importance of incorporating media and different models in the teaching and learning process as also previously highlighted (Aris, Putri, & Susanti, 2017; Sen, Tengah, Shahrill & Leong, 2017; Steward, n.d.). These allowed the participants to visualize the concept and thus making understanding easier. The use of real objects such as money made it easier to graduate the learners from concrete objects to conceptual and abstract understanding. However, the counters and the number line had limitations on the shift from concrete to abstract, as the learner is supposed to make imaginations, which becomes difficult to teach and learn.

The strategies and techniques used could only utilize integers neglecting the fact that the concept of subtraction of directed numbers is also applicable to fractions and decimals. The media used could not allow the demonstration of such and the teacher left such concepts. The strategies only covered the arithmetic part of the directed numbers and neglected the algebraic part where the concept of directed numbers is also applicable (Steward, n.d., Wessman-Enzinger, & Mooney, 2019).

The results from the counters strategy indicated that the teacher needed to move away from assuming that certain techniques will not be comprehended by the learners. The teacher had initially thought the counters model would be too difficult for the learners, but however, the results indicated otherwise.

The ability of this action research to solve the participants' problems reflected to us the need and the importance of going beyond the classroom limits to solve teaching and learning problems. In turn, this enhances teaching practices.

The results from the cooperative learning strategy and the counters model highlighted to us the importance and benefits of varied approaches to the teaching and learning process. The cooperative learning strategy highlighted the possibility of enhanced learning through group and peer learning. The use of one-on-one approach during the implementation of the counters model indicated the importance of utilizing such techniques, especially in remediation.

In the first cycle, the teacher gave all the four participants the same tasks because at the beginning she noticed that the participants had a common problem. However, at the end of Cycle 1 participant D emerged to have more serious problems than the other participants. It was apparent that the teacher needed to consider individual differences in the intervention activities.

This research highlights a lot of lessons about teaching practices and the teacher-researchers, in this particular case, will be helped and challenged to continuously improve their own practices and enhance their professional development.

Limitations

Although the students' failure to subtract directed numbers were solved in this research, there were some limitations which could threaten the trustworthiness of the results. One major limitation was that of time. The time to meet with the participants was limited due to their packed timetable. Normally, for the purposes of this research, the researchers met the participants after 1600 hours, for an average time of one hour. At this hour, the participants would already be tired after a long day, notwithstanding that most learners prefer learning mathematics in the morning. At the end of the day the participants would have minimum concentration. Different results could be yielded if the research was to be done during the early hours of the day where learner participation is expected to be at maximum.

One of the participants in this research was a day scholar, coming from a distant village, which forced the teacher to rush in her activities so that she could release the participant on time, to allow the student time to safely walk home. Another limitation was that the researchers had to conduct the research at the same time carrying out other duties elsewhere, and so had to budget time and resources, among other things.

CONCLUSION

The purpose of this action research was to solve Form 2 students' failure to subtract directed numbers. The study had two cycles. Cycle 1 had three sessions and four participants. In cycle 1 the teacher utilized the cooperative learning strategy together with the number line and the having-money-owing money techniques. Using this strategy, three participants managed to grasp the concept of subtraction of directed numbers. Cycle 2 had the one participant who had failed to grasp the concept using the cooperative strategy. The cycle had two sessions and utilized the counters model/ strategy. The strategy was effective in solving the participant's failure to subtract directed numbers.

As indicated from the reflections and limitations, the study can be extended to incorporate fractions, decimals and algebraic expressions for it to cover other important and examinable areas. We feel convinced that our research study is an eye-opener for further study by other researchers who may experience similar computational problems with directed numbers among the learners in different contexts.

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