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Mathematical reasoning in quadratic functions: systematic literature review of challenges, instruction, assessment, and future directions

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Abstract

Mathematical reasoning is central to meaningful learning of quadratic functions, yet research on how students reason about this topic remains fragmented across studies of representations, misconceptions, and instructional design. This systematic review synthesizes international evidence on mathematical reasoning in quadratic functions, addressing students' challenges, instructional approaches, assessment indicators, and future research directions. The review followed PRISMA 2020 but searched only Google Scholar for studies published between January 2021 and August 2026; the exact search date was not logged, and both are noted as limitations. From an initial 80 records, 62 proceeded to title and abstract screening, 29 full-text reports were assessed against topic, design, and journal-quality eligibility criteria detailed in the Method, and 10 empirical studies were retained (8 on quadratic functions, 2 on quadratic equations). The included studies involved students from Grades VI through XI and prospective teachers across six countries, using predominantly qualitative designs, with two studies involving only two student pairs. Findings suggest that students' difficulties center on conceptual understanding, prerequisite algebraic knowledge, coordinating multiple representations, covariational reasoning, generalization, and justification, although several themes are based on only one or two studies and are illustrative rather than established. Instructional approaches combined multiple representations, figural-pattern generalization, contextual and collaborative learning, and teacher mediation, while assessment increasingly treated reasoning as an observable process rather than only a final answer. Given the small, single-database, heterogeneous evidence base, the review offers a provisional synthesis and research agenda rather than generalizable claims, pointing to larger samples, multi-database searches, integrated conceptual frameworks, and technology-mediated learning.

Keywords: Covariational Reasoning; Mathematical Reasoning; Multiple Representations; Quadratic Functions; Systematic Literature Review.

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INTRODUCTION

International large-scale assessments indicate that many students still struggle to reason mathematically beyond routine procedures: in the most recent Programme for International Student Assessment (PISA) 2025 cycle, Indonesian 15-year-olds obtained a mean mathematics score of 364 points, nearly 100 points below the 2025 OECD average of 463, (OECD, 2023, 2026). Mathematics education is undergoing a global shift from transmission-oriented

instruction toward learning environments that cultivate mathematical reasoning, problem solving, modelling, technology use, and meaningful participation. International research agendas identify teaching approaches, educational goals, technology, equity, and assessment as persistent priorities for the development of mathematics education during the coming decade (Bakker et al., 2023). These developments make mathematical reasoning particularly important because mathematics learning involves not only obtaining correct answers but also constructing, evaluating, communicating, and justifying mathematical claims (NCTM, 2000; Lithner, 2008). These broader difficulties with non-routine mathematical reasoning are not confined to large-scale assessments; they also surface within specific mathematical domains, quadratic functions prominent among them.

Quadratic functions constitute an important domain for investigating mathematical reasoning because learning this topic requires students to coordinate symbolic, graphical, numerical, verbal, and contextual representations while reasoning about variation, parameters, roots, extrema, and nonlinear relationships (Ramírez et al., 2022; Wilkie, 2024). Evidence from Spain shows that students working with quadratic functional relationships can recognize relationships and employ multiple representations, but they experience difficulties expressing quadratic structures symbolically and generalizing them accurately (Ramírez et al., 2022).

Research in Australia similarly found that secondary students' work with quadratics was constrained by weak prerequisite algebraic knowledge, misconceptions concerning the null-factor law, and curriculum timeframes that limited the development of conceptual understanding and procedural fluency (Reid O'Connor & Norton, 2024). A qualitative study involving Year 10 students demonstrated that figural-pattern generalization can create opportunities for coordinating visual and algebraic reasoning and attending to important quadratic-function concepts (Wilkie, 2024). At the international level, a recent systematic review of 38 empirical studies reported that students' difficulties with quadratic equations were frequently associated with insufficient prerequisite knowledge, inflexible knowledge, non-relational understanding, and instructional contexts that separated procedures from concepts (Gustafsson, 2026). In Indonesia, design-based studies developing instructional materials for quadratic equations and functions have similarly shown that structured student worksheets emphasizing problem solving and Toulmin-based argumentation can support students' mathematical reasoning and argumentation skills (Meriza et al., 2022; Putri et al., 2024).

Despite these contributions, a sharp research gap remains in the literature on mathematical reasoning in quadratic functions. First, recent quadratic-function studies remain

fragmented across representations, misconceptions, curriculum structures, problem solving, and technology rather than explicitly synthesizing mathematical reasoning as the central analytical construct (Ramírez et al., 2022; Reid O'Connor & Norton, 2024; Wilkie, 2024). Second, although (Gustafsson, 2026) systematic review provides an important synthesis of empirical research on quadratic equations across multiple countries, its scope deliberately concentrates on quadratic equations and equation-solving rather than the broader reasoning processes involved in quadratic functions. Third, research on mathematical reasoning has developed diverse analytical frameworks, including conjecturing, generalizing, justifying, creative reasoning, and diagnostic reasoning, but these frameworks have not been systematically mapped to the specific conceptual demands of quadratic functions (Berge & Solberg, 2026; Säfström et al., 2024; Widjaja et al., 2021). Consequently, the field lacks an integrated evidence base that explains how students' challenges, instructional responses, assessment indicators, and future research priorities are interconnected within the specific domain of quadratic functions (Cevikbas et al., 2024).

This study addresses this gap through a systematic literature review that conceptualizes mathematical reasoning in quadratic functions as a multidimensional research object rather than as a single achievement score. The contribution of the review lies in an integrative thematic synthesis of four analytical dimensions, students' challenges, instructional approaches, assessment indicators, and future research directions, specific to quadratic functions, extending previous reviews that have predominantly emphasized either quadratic equations or broader mathematics-education trends (Cevikbas et al., 2024; Gustafsson, 2026). The review also distinguishes content-specific difficulties in quadratic functions from broader reasoning difficulties, enabling more precise interpretation of whether reported problems concern conceptual understanding, representation coordination, justification, generalization, strategy selection, or the relationship between procedures and concepts (Ramírez et al., 2022; Säfström et al., 2024). From a theoretical perspective, the review connects domain-specific reasoning in quadratic functions with contemporary conceptualizations of mathematical reasoning, while from a practical perspective it seeks to generate evidence that can inform curriculum design, instructional development, assessment construction, and technology-enhanced mathematics learning (Salam et al., 2023; Weigand et al., 2024).

This study therefore aims to synthesize recent international evidence on mathematical reasoning in quadratic functions and to identify implications for mathematics education research and practice. Specifically, the review addresses four research questions: (1) What

challenges do students experience in developing mathematical reasoning about quadratic functions? (2) What instructional approaches have been implemented to enhance students' mathematical reasoning in quadratic functions? (3) What indicators are used to assess students' mathematical reasoning in studies on quadratic functions? and (4) what future research directions are reported by the authors of the included studies, or can be reasonably inferred from recurring patterns across them, regarding mathematical reasoning in quadratic functions?

METHODS

This study employed a focused Systematic Literature Review (SLR) to identify, appraise, and synthesize empirical evidence on mathematical reasoning in quadratic functions, focusing on students' challenges, instructional approaches, assessment indicators, and future research directions. The review was reported according to the PRISMA 2020 framework to ensure transparency in study identification, screening, eligibility assessment, and inclusion.

The literature search was conducted using Google Scholar as the primary search source for studies published from January 2021 to August 2026. The core search strategy combined the concepts of mathematical reasoning and quadratic mathematical content using the expression ("mathematical reasoning" OR "reasoning ability") AND ("quadratic function" OR "quadratic equation"). The search was restricted to English-language scholarly publications relevant to mathematics education.

Google Scholar was selected as the sole search source because it offers broad cross-disciplinary, including regionally published outlets not consistently captured by a single subscription database. The search was executed once, using the Boolean string reported above, restricted to English-language results and the publication window of January 2021-August 2026, with no additional Google Scholar filters applied. The exact calendar date of this search was not recorded. The unit of analysis consisted of empirical peer-reviewed journal articles addressing mathematical reasoning in quadratic functions or closely related quadratic concepts. The inclusion and exclusion criteria are presented in Table 1.

Journal indexing and quality status were verified using the Scimago Journal Rank (SJR) database. A journal was considered to meet the indexing/quality threshold if it had been assigned an SJR quartile ranking (Q1-Q4) at any point in its publication history, regardless of whether that ranking was subsequently continued or discontinued at the time of verification; journals with no SJR-listed quartile ranking at any point were excluded at the eligibility stage.

Because mathematical reasoning is theorized through multiple complementary constructs in the literature, a study was classified as addressing mathematical reasoning if it explicitly investigated at least one of the following recognized sub-constructs: covariational reasoning (coordinating the joint variation of two or more quantities), functional/relational reasoning (interpreting a mathematical object as a relationship between covarying quantities), representational reasoning (constructing, interpreting, or coordinating multiple representations of the same relationship), generalisation (extending regularities observed in particular cases into general mathematical statements), justification/argumentation (constructing or evaluating the validity of a mathematical claim), visual reasoning (identifying or interpreting mathematical structure from visual or figural representations), and intuitive-reflective reasoning (the interaction between rapid, heuristic responses and deliberate, analytic validation). Borderline reports for instance, studies emphasising procedural error analysis or routine algebraic manipulation were retained only when the authors' own analytic framework explicitly linked the reported student behaviour to one or more of these sub-constructs; procedural correctness or computational accuracy alone was not treated as sufficient evidence of mathematical reasoning.

Table 1. Eligibility Criteria for Study Selection

Criterion	Inclusion	Exclusion
Publication year	2021-August 2026	Outside the period
Language	English	Non-English
Document type	Peer-reviewed journal article	Thesis, book, proceedings, report, etc.
Research type	Empirical study	Theoretical/non-empirical study
Mathematical content	Quadratic function, quadratic equation, parabola, or related quadratic representation	Unrelated mathematical content
Reasoning focus	Mathematical reasoning or related reasoning constructs	No substantive reasoning focus
Context	Mathematics education/learning	Non-mathematics education context
Quality/indexing	Meets predefined indexing/quality criteria	Does not meet the criteria
Relevance	Addresses at least one RQ	Does not contribute to any RQ

The initial Google Scholar search yielded 80 records. Before screening, 18 records were removed, consisting of duplicates (n = 12), publications outside 2021-2026 (n = 4), non-English records (n = 1), and non-journal documents (n = 1), leaving 62 records for title and abstract screening. Thirty-three records were excluded because they were unrelated to quadratic functions (n = 20), mathematical reasoning (n = 9), or the mathematics education/learning context (n = 4). The remaining 29 full-text reports were retrieved and assessed for eligibility. Nineteen reports were subsequently excluded because their journals did not meet the predefined indexing/quality threshold (n = 6), their outcome or domain did not explicitly correspond to

mathematical reasoning and quadratic functions (n = 8), they were previous systematic reviews or meta-analyses (n = 3), or they provided insufficient empirical evidence (n = 2). Consequently, 10 studies were included in the final qualitative synthesis. Of these ten, eight studies focus primarily on quadratic functions and two focus on quadratic equations (Baybayon & Lapinid, 2024; Reid O'Connor & Norton, 2024); the two domains are mathematically related, so both were retained, but findings from the two equation-focused studies are reported separately where they diverge from the function-focused majority (see Limitations).

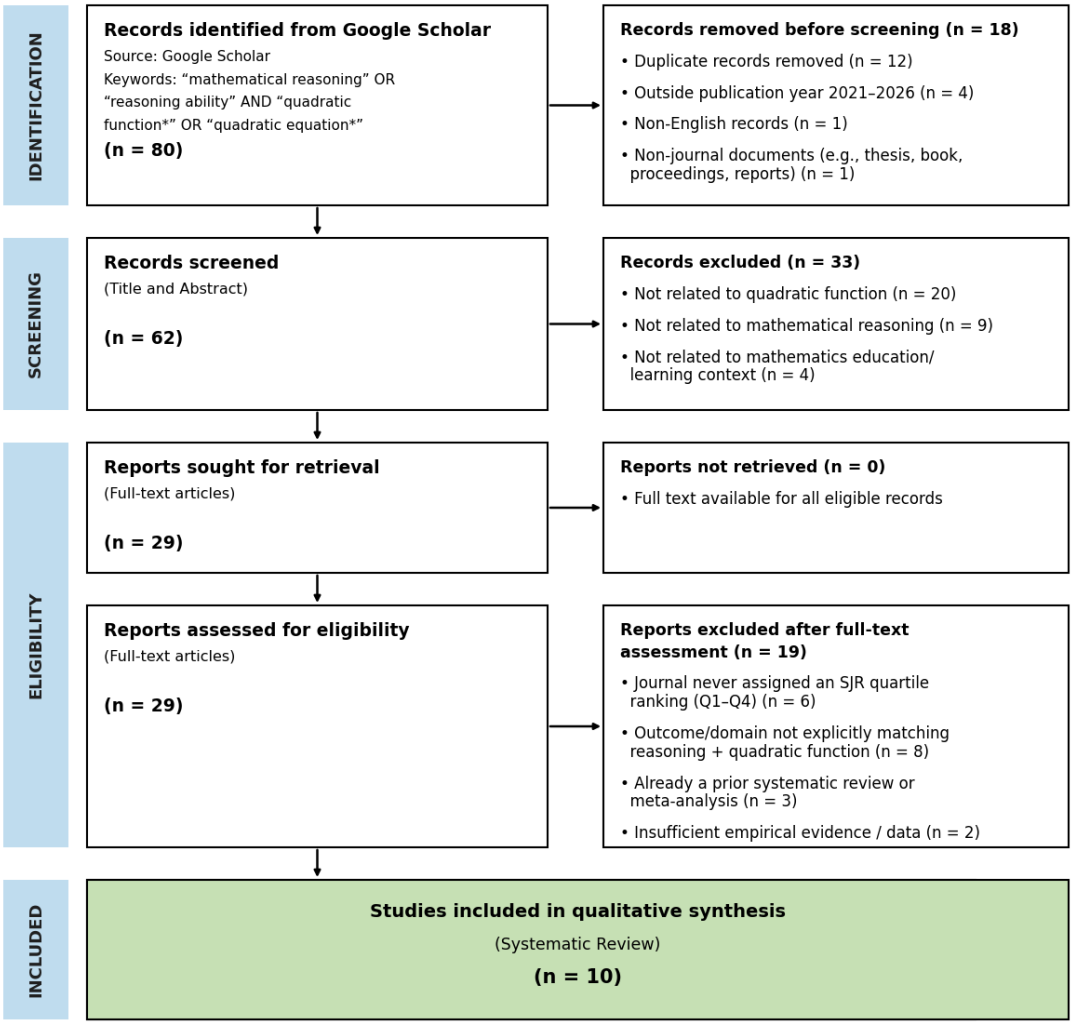


Figure 1. PRISMA 2020 flow diagram of the study selection process (Page et al., 2021).

Data were collected using a search protocol, structured data-extraction form, and quality-appraisal checklist. The extraction form recorded author(s), year, country, journal, participants, educational level, research design, quadratic-function focus, mathematical reasoning construct, instructional context, assessment approach, limitations, research gaps, and reported contributions.

Each of the ten included studies was appraised by the first author, with disagreements resolved through discussion with the second author, against five criteria rated qualitatively as Adequate (A) or Partially Adequate (PA): (1) clarity of research objectives, (2) appropriateness of design, (3) adequacy of participant/context description, (4) transparency of data collection and analysis, and (5) consistency between findings and conclusions. This checklist was applied descriptively rather than as a second exclusion gate, to characterise methodological strengths and limitations and moderate the review's conclusions; no study was excluded on this basis. These ratings, summarised in Table 2, informed the moderated interpretation of the review's conclusions in the Discussion.

Table 2. Quality Appraisal Results for the Included Studies

No	Study	Objectives	Design	Participants/ Context	Data Transparency	Consistency
1	Hohensee et al. (2021)	A	A	A	PA	A
2	Wilkie (2022)	A	A	PA[1]	PA	A
3	Ramírez et al. (2022)	A	A	A	PA	A
4	Rahmawati & Putri (2022)	A	A	PA	PA	PA
5	Baybayon & Lapinid (2024)	A	A	A	PA	A
6	Reid O'Connor & Norton (2024)	A	A	A	PA	A
7	Wilkie (2024)	A	A	PA[1]	A	A
8	Abdelmalak (2025)	A	A	A	PA	PA
9	Altindis & Fonger (2025)	A	A	A	PA	PA
10	Puspitasari et al. (2026)	A	A	PA	PA	A

Note. A = Adequate; PA = Partially Adequate (no study rated Inadequate or excluded on this basis). [1] Rated Partially Adequate on sample-size adequacy due to the case-based design involving two student pairs.

The extracted data were analyzed through descriptive analysis and qualitative thematic synthesis. Descriptive analysis summarized publication year, country, educational level, participants, research design, quadratic content, reasoning construct, instructional context, and assessment indicators. Thematic synthesis was organized deductively around the four research questions and inductively through recurring patterns across the included studies. Microsoft Excel was used to organize screening records, study characteristics, extraction data, and thematic coding. Because the included studies differed substantially in participants, designs, constructs, instruments, and outcomes, no meta-analysis or inferential statistical test was conducted; instead, findings were synthesized qualitatively.

RESULTS AND DISCUSSION

Characteristics of the Included Studies

The systematic selection process resulted in 10 studies included in the qualitative synthesis. As shown in Figure 1, the search through Google Scholar initially identified 80 records, of which 62 proceeded to title and abstract screening, 29 full-text reports were assessed for eligibility, and 10 studies were finally retained. The included studies represent six countries: Australia, the United States, Indonesia, Egypt, the Philippines, and Spain. Australia contributed three studies, the United States and Indonesia contributed two studies each, while Egypt, the Philippines, and Spain contributed one study each.

The participants and educational contexts varied substantially. The studies included Grade VI, Grade IX, Grade X, and Grade XI students, prospective middle-school teachers, and secondary students in different instructional settings. Sample sizes ranged from two students in an Indonesian qualitative study to 57 students in the study by Hohensee et al. (2021), while other studies involved paired students, 22 Grade IX students, 46 Grade 9 students, or 25 Year 11 students. This variation provides evidence across different stages of mathematical learning but also indicates that much of the existing evidence remains based on relatively small and context-specific samples.

Methodologically, the evidence base is dominated by qualitative and descriptive approaches, including qualitative case studies, descriptive-exploratory studies, teaching experiments, classroom-interaction analyses, and qualitative interventions. The quadratic content was predominantly quadratic functions, although two studies explicitly addressed quadratic equations. Quadratic equations were retained within the eligibility criteria alongside quadratic functions because, conceptually, an equation of the form $ax^2 + bx + c = 0$ is a special case of the quadratic function $f(x) = ax^2 + bx + c$ under the condition $f(x) = 0$, and the two studies addressing equations specifically Baybayon & Lapinid (2024) and Reid O'Connor & Norton (2024) examined reasoning about roots, factoring, and the null-factor law, which are properties of the same underlying quadratic relationship examined functionally elsewhere in the corpus (e.g., through graphs, tables, and covariation). To avoid conflating these two related but distinct problem spaces, the two equation-focused studies are named explicitly wherever they are a finding's primary source (e.g., in the traceability note beneath Table 4); this attribution is not a systematic tag on every claim (see Limitations). The reasoning constructs were heterogeneous, including covariational reasoning, representational fluency, visual reasoning, algebraic reasoning, functional thinking, generalisation, intuitive-reflective reasoning, strategic reasoning, and justification.

This heterogeneity suggests that mathematical reasoning in quadratic functions is not yet represented by a single dominant theoretical or operational framework. Instead, the literature approaches reasoning through complementary lenses. For example, Hohensee et al. (2021) focused on covariational reasoning and backward transfer, Wilkie (2022, 2024) emphasized noticing, visualization, and generalisation, Ramírez et al. (2022) examined functional thinking and representations, while Puspitasari et al. (2026) used a dual-process perspective to distinguish intuitive and reflective reasoning. Thus, the literature provides substantial conceptual resources but remains fragmented in how mathematical reasoning is defined and observed in quadratic functions.

Table 3. Characteristics of the Included Studies

No.	Author(s), Year	Country	Journal	Design	Participants
1	Hohensee et al. (2021)	United States	Journal of Mathematical Behavior	Exploratory quasi-experimental (pre-/post-assessment)	57 students, two algebra classrooms
2	Wilkie (2022)	Australia	Journal of Mathematical Behavior	Fine-grained qualitative task-based study	Two pairs of Year 10 students
3	Ramírez et al. (2022)	Spain	ZDM-Mathematics Education	Qualitative/descriptive (questionnaire with geometric tasks)	Grade VI (6th-grade) students
4	Rahmawati & Putri (2022)	Indonesia	AKSIOMA: Jurnal Program Studi Pendidikan Matematika	Descriptive qualitative (PMRI + collaborative learning)	Junior/senior secondary students
5	Baybayon & Lapinid (2024)	Philippines	Infinity	Descriptive qualitative (error analysis)	46 Grade IX students
6	Reid O'Connor & Norton (2024)	Australia	Mathematics Education Research Journal	Qualitative/descriptive study	25 Year 11 students
7	Wilkie (2024)	Australia	Mathematics Education Research Journal	In-depth qualitative task-based study	Two pairs of Year 10 students
8	Abdelmalak (2025)	Egypt	Research in Mathematics Education	TMSSR-based intervention (scriptwriting tasks)	17 prospective (middle-school) teachers
9	Altindis & Fonger (2025)	United States	Education Sciences	Teaching experiment (community-center setting)	8 Turkish–American secondary students
10	Puspitasari et al. (2026)	Indonesia	Journal of Advanced Sciences and Mathematics Education	Qualitative descriptive (dual-process perspective)	Grade X students

Mathematical Reasoning Embedded in Quadratic Functions

Before turning to students' challenges, instructional approaches, and assessment practices, it is useful to identify what forms of mathematical reasoning are intrinsically demanded by the content of quadratic functions themselves. Across the included studies, quadratic functions consistently required students to engage in covariational reasoning, tracking

how one quantity changes in relation to another as they investigated non-constant rates of change (Altindis & Fonger, 2025; Hohensee et al., 2021). The topic also demanded functional and relational thinking, whereby students interpreted a quadratic function as a relationship between covarying quantities rather than as an isolated symbolic expression (Ramírez et al., 2022). A further recurring form of reasoning involved coordinating multiple representations connecting symbolic, graphical, tabular, and contextual depictions of the same quadratic relationship, and noticing which features of these representations were mathematically significant (Wilkie, 2022, 2024). Generalisation was similarly central: several studies examined how students extended patterns observed in specific quadratic figural or numerical sequences into general symbolic rules (Wilkie, 2022). Other studies highlighted intuitive versus reflective reasoning, distinguishing rapid, heuristic responses to quadratic problems from more deliberate, analytic reasoning (Puspitasari et al., 2026), as well as justification and argument validity, in which students constructed and evaluated the soundness of mathematical claims about quadratic relationships (Rahmawati & Putri, 2022). Finally, several studies pointed to strategic and procedural-conceptual reasoning, in which students needed to coordinate algebraic procedures with an understanding of the underlying quadratic concept (Baybayon & Lapinid, 2024; Reid O'Connor & Norton, 2024). These reasoning forms, together, indicate that quadratic functions are not merely a topic to be learned procedurally, but a mathematically rich domain that inherently requires covariational, representational, generalising, justificatory, and strategic reasoning. This reasoning-centred view of the content itself provides the conceptual basis for examining, in the following sections, the challenges students encounter, the instructional approaches used to support this reasoning, and how it has been assessed.

Students' Challenges in Developing Mathematical Reasoning (RQ1)

The synthesis points to six interconnected challenge areas, summarized in Table 4: conceptual understanding, prerequisite algebraic knowledge, representation coordination, covariational reasoning, generalisation, and justification. These difficulties rarely occur in isolation; students most often struggle when several of these demands converge at once, such as moving between representations, identifying underlying relationships, and formulating general claims.

Table 4. Students' Challenges in Developing Mathematical Reasoning

No.	Challenge Theme	Specific Challenge	Evidence Across the Included Studies	Reasoning Dimension	Supporting Studies
1	**Conceptual understanding**	Difficulty developing conceptual understanding of	Students experience difficulties when quadratic concepts require	Conceptual reasoning	2

No.	Challenge Theme	Specific Challenge	Evidence Across the Included Studies	Reasoning Dimension	Supporting Studies
		quadratic structures and relationships	understanding beyond procedures and formulas		
2	**Prerequisite algebraic knowledge**	Weak prerequisite knowledge interferes with quadratic reasoning	Difficulties with prerequisite algebraic knowledge contribute to errors and difficulties in quadratic learning	Algebraic reasoning	2
3	**Representation coordination**	Difficulty connecting visual, symbolic, graphical, and other representations	Students need to coordinate multiple representations to identify and express quadratic structures	Representational reasoning	4
4	**Visual–algebraic coordination**	Difficulty translating visual structures into algebraic forms	Figural-pattern tasks reveal challenges in coordinating visual and algebraic reasoning	Visual and algebraic reasoning	2
5	**Generalisation**	Difficulty identifying and expressing general quadratic structures from patterns	Students may identify patterns but experience challenges when extending them into general relationships	Generalising	3
6	**Covariational reasoning**	Difficulty coordinating changing quantities and relationships	Quadratic instruction requires students to reason about relationships between changing quantities	Covariational reasoning	2
7	**Procedural dependence**	Reliance on procedures without sufficient conceptual reasoning	Difficulties arise when students use procedures without adequately explaining or understanding the underlying relationships	Strategic/procedural reasoning	2
8	**Justification and validation**	Difficulty explaining, validating, or defending mathematical strategies	Reasoning requires students to provide explanations and evaluate the validity of mathematical arguments	Justification/argumentation	2
9	**Noticing relevant mathematical structures**	Difficulty identifying mathematically significant features during pattern exploration	Student noticing influences what aspects of quadratic figural patterns become objects of reasoning	Noticing/visual reasoning	1
10	**Intuitive–reflective coordination**	Difficulty coordinating intuitive responses with reflective validation	Quadratic problem solving may require students to move beyond intuitive responses toward reflective reasoning	Intuitive–reflective reasoning	1

Note on thematic traceability. To make the evidentiary basis of Table 4 explicit, the specific supporting studies for each theme are as follows: (1) conceptual understanding-Rámirez et al. (2022) and Reid O'Connor & Norton (2024); (2) prerequisite algebraic knowledge- Baybayon & Lapinid (2024) and Reid O'Connor & Norton (2024); (3) representation coordination-Altindis & Fonger (2025), Ramírez et al. (2022), Wilkie (2022), and Wilkie (2024); (4) visual algebraic coordination-Wilkie (2022) and Wilkie (2024); (5) generalisation-Wilkie (2022), Wilkie (2024), and Ramírez et al. (2022); (6) covariational reasoning-Hohensee et al. (2021) and Altindis & Fonger (2025); (7) procedural dependence-Baybayon & Lapinid (2024) and Reid O'Connor & Norton (2024); (8) justification and validation-Rahmawati & Putri (2022) and Altindis & Fonger (2025); (9) noticing relevant mathematical structures-Wilkie (2022) only; and (10) intuitive-reflective coordination-Puspitasari et al. (2026) only. Themes 9 and 10 are each supported by a single study and are accordingly interpreted as illustrative rather than well-established patterns in the Discussion.

These challenges cluster around three interrelated issues. Coordinating multiple representations is one: Altindis & Fonger (2025) found that students had to connect

covariational reasoning with representational fluency when working with contextually grounded tasks, while Wilkie (2022, 2024) showed that students' reasoning in figural-pattern tasks depends on what they notice in visual representations and how they connect those observations with algebraic forms (the 2022 study identified 16 Centres of Focus illustrating how different visual features become objects of attention). Closely related is generalisation: Wilkie's studies demonstrate that students must move from noticing particular visual features toward general algebraic relationships, connecting visualisation, figural patterns, and algebraic reasoning. A third issue concerns prerequisite knowledge and conceptual–procedural coordination: Baybayon & Lapinid (2024); Reid O'Connor & Norton (2024) both linked persistent errors and misconceptions in quadratic equations to weaknesses in prerequisite algebraic knowledge, curriculum structure, and the coordination of conceptual understanding with procedural fluency.

Taken together, these findings suggest that quadratic reasoning difficulties are not reducible to weak manipulation of formulas; the more fundamental issue is whether students can coordinate quantities, representations, structures, and explanations at the same time. Quadratic functions demand exactly this kind of coordination: symbolic, graphical, numerical, geometric, and contextual forms all have to be held together, which is consistent with the representation-coordination and conceptual-understanding difficulties identified in Table 4 (Altindis & Fonger, 2025; Ramírez et al., 2022; Reid O'Connor & Norton, 2024).

Instructional Approaches and Features Reported in the Studies (RQ2)

Table 5 catalogues the instructional features reported across the included studies: multiple representations, figural-pattern generalisation, contextual learning, collaborative learning, teacher mediation, formative feedback, and problem-solving. No single instructional model dominated the corpus.

Table 5. Instructional Approaches and Features Reported in the Included Studies

No.	Instructional Approach	Main Instructional Features	Reasoning Target	Evidence from Included Studies	Studies
1	**Quadratic-function instruction**	Multi-lesson instruction focusing on quadratic relationships and connections with previously learned linear functions	Covariational reasoning; transfer	Quadratic instruction was associated with both enhancing and undermining changes in students' prior covariational reasoning about linear functions.	Hohensee et al. (2021)
2	**Multiple representations**	Coordinating visual, graphical, tabular, and algebraic representations	Representational fluency; visual-algebraic reasoning	Students worked with multiple visualisations and representations to connect quadratic	Altindis & Fonger (2025); Wilkie (2024);

No.	Instructional Approach	Main Instructional Features	Reasoning Target	Evidence from Included Studies	Studies
				structures with algebraic forms.	Ramírez et al. (2022)
3	**Figural pattern generalisation**	Growing-pattern exploration, creation, comparison, generalisation, and movement between patterns and algebraic expressions	Visual reasoning; generalisation; algebraic reasoning	Figural patterns were used to support students in identifying quadratic structures and connecting visual patterns with general algebraic forms.	Wilkie (2022, 2024)
4	**Culturally inclusive learning environment**	Contextual tasks, peer dialogue, cultural practices, and multiple representations	Covariational reasoning; representational fluency	The learning environment integrated contextual and culturally responsive interactions with multiple representations.	Altindis & Fonger (2025)
5	**PMRI + collaborative learning**	Realistic contexts, collaboration, sharing and jumping tasks, and video-supported learning	Mathematical reasoning; mathematical manipulation; argument validation	PMRI was combined with collaborative learning and video to provide opportunities for students to develop and communicate mathematical reasoning.	Rahmawati & Putri (2022)
6	**Teacher moves for supporting reasoning**	Eliciting, responding to, facilitating, and extending student reasoning through TMSSR-based intervention	Student reasoning; generalisation; justification	The intervention focused on developing prospective teachers' use of high-potential teacher moves to support students' reasoning on quadratic functions.	Abdelmalak (2025)
7	**Blended/differentiated learning with formative feedback**	Tiered worksheets, differentiated learning activities, formative assessment, and immediate feedback	Error correction; prerequisite knowledge; procedural/conceptual reasoning	Tiered worksheets and immediate feedback were used to identify and address persistent errors in quadratic equations.	Baybayon & Lapinid (2024)
8	**Problem-solving with intuitive-reflective processes**	Quadratic-function problem solving followed by examination and justification of reasoning processes	Intuitive-reflective reasoning; strategy evaluation	Students' problem-solving processes were examined in terms of how intuitive and reflective reasoning interacted when solving quadratic-function problems.	Puspitasari et al. (2026)
9	**Geometric/contextual representation**	Geometric situations and multiple representations to explore quadratic structures	Functional thinking; representation; generalisation	Geometric contexts and multiple representations supported students' exploration of quadratic structures and generalisation.	Ramírez et al. (2022)

Note. Instructional features were not uniformly tested for effectiveness. Only Hohensee et al. (2021) employed a pre-/post-assessment design; the remaining evidence was primarily qualitative or descriptive.

Two broad instructional features recur across the corpus. Multiple representations were most prominent: Altindis & Fonger (2025) situated quadratic-function tasks within a culturally inclusive learning environment to examine the co-development of covariational reasoning and

representational fluency, while Ramírez et al. (2022; Wilkie (2022) similarly used multiple visualisations and representations to support structures and figural-pattern generalisation (19 tasks examining how pair interaction and task design influenced students' noticing and generalising). Social and teacher-mediated dimensions were also important: Rahmawati & Putri (2022) combined PMRI, collaborative learning, video, and sharing-and-jumping tasks to support reasoning, manipulation, and argument validity; Abdelmalak (2025) focused on prospective teachers' use of high-potential teacher moves (eliciting, responding to, facilitating, and extending student reasoning); and Baybayon & Lapinid (2024) used differentiated worksheets, formative assessment, and immediate feedback to address persistent errors and prerequisite knowledge.

The instructional evidence suggests that mathematical reasoning is supported less by a single named pedagogy than by a set of instructional affordances rich representations, cognitively demanding tasks, opportunities for generalisation, dialogue, feedback, and teacher mediation. Read this way, the useful question stops being which instructional model is "best" and becomes which features of a learning environment actually open up space for reasoning.

However, a notable limitation is that the included studies provide limited evidence concerning dynamic mathematics technology such as dynamic geometry software (e.g., GeoGebra) or dynamic graphing tools (e.g., Desmos) as an explicit mechanism for supporting reasoning in quadratic functions. The literature strongly emphasizes visualisation and representation, but the final 10-study corpus does not establish any such tool as a tested instructional approach. Therefore, the role of dynamic mathematics/graphing technology should be regarded as a future research opportunity, rather than as an established finding of the current evidence base.

Assessment Indicators of Mathematical Reasoning (RQ3)

Table 6 shows that the assessment indicators reported across the studies were multidimensional, spanning covariational reasoning, representational fluency, visual reasoning, noticing, generalisation, functional thinking, justification, argument validation, strategic reasoning, and intuitive-reflective reasoning.

Table 6. Assessment Indicators of Mathematical Reasoning in Quadratic Functions

No.	Reasoning Dimension	Assessment Indicator	Observable Evidence	Included Studies
1	**Covariational reasoning**	Coordinating two or more changing quantities and describing how they change in relation to one another	Students' written explanations, comparisons of quantities, and pre-/post-assessment responses	Hohensee et al. (2021); Altindis & Fonger (2025)

			showing the quantities considered in reasoning	
2	**Representational fluency**	Constructing, interpreting, and coordinating multiple representations of quadratic relationships	Use and connection of tables, graphs, symbolic expressions, and contextual representations to explain mathematical relationships	Altindis & Fonger (2025); Ramírez et al. (2022); Wilkie (2024)
3	**Visual reasoning**	Identifying and interpreting mathematical structures or relationships from visual/figural representations	What students notice in figures, diagrams, patterns, and visualisations and how these observations support mathematical reasoning	Wilkie (2022, 2024)
4	**Generalisation**	Identifying regularities and extending particular cases into a general mathematical relationship	General rules, symbolic expressions, equivalent forms, and explanations derived from quadratic figural patterns	Wilkie (2022, 2024); Ramírez et al. (2022)
5	**Functional/structural reasoning**	Recognising relationships, structures, and dependencies underlying quadratic situations	Identification of functional structures and relationships across numerical, pictorial, geometric, and symbolic representations	Ramírez et al. (2022)
6	**Mathematical justification**	Explaining why a strategy, representation, or mathematical claim is valid	Verbal/written explanations, comparison of mathematical models, justification of choices, and responses to questions or challenges	Altindis & Fonger (2025); Rahmawati & Putri (2022)
7	**Argumentation and validation**	Evaluating and defending the validity of mathematical reasoning or arguments	Students' ability to check, explain, critique, and validate mathematical arguments and solutions	Rahmawati & Putri (2022); Altindis & Fonger (2025)
8	**Strategic reasoning**	Selecting, explaining, and evaluating solution strategies rather than relying solely on procedures	Written solutions, diagnostic interviews, explanations of strategies, and analysis of conceptual versus procedural reasoning	Reid O'Connor & Norton (2024)
9	**Intuitive–reflective reasoning**	Moving from an initial/intuitive response toward reflective examination and validation	Written problem-solving responses, video-recorded reasoning processes, and interviews used to examine System 1-System 2 interaction	Puspitasari et al. (2026)
10	**Noticing and reasoning processes**	Identifying mathematically productive features and explaining their relevance to generalisation	Centres of Focus inferred from verbal utterances, gestures, written inscriptions, and student interactions	Wilkie (2022, 2024)
11	**Reasoning-support indicators**	Eliciting, responding to, facilitating, and extending students' mathematical reasoning	Teacher scripts and responses showing elicitation of understanding, error correction, guidance, reflection, and pressing for generalisation	Abdelmalak (2025)
12	**Conceptual/procedural understanding**	Distinguishing conceptual understanding from procedural performance and identifying persistent misconceptions	Written tests, interviews, error analysis, worksheets, and formative assessment evidence	Baybayon & Lapinid (2024); Reid O'Connor & Norton (2024)

Note. Row 11 represents teacher reasoning-support practices rather than direct indicators of students' reasoning. Assessment instruments are described in the Observable Evidence column; no common scoring rubric was identified across the included studies.

Covariational reasoning, representation, and generalisation were the most prominent indicators. Hohensee et al. (2021) examined changes in the number of quantities students considered when reasoning about linear-function situations before and after quadratic-function instruction, interpreting changes as potential evidence of backward-transfer influences, while Altindis & Fonger (2025) emphasized the relationship between covariational reasoning and representational fluency in contextual tasks. Wilkie (2022) used students' verbal utterances, gestures, and written inscriptions to identify Centres of Focus during figural-pattern generalisation, and Ramírez et al. (2022) examined structures, representations, and generalisation in students' work with quadratic functions. Justification and validation, strategic reasoning, and conceptual-procedural understanding were examined through explanations, written tests, interviews, and error analysis across the remaining studies.

A central finding is that mathematical reasoning was generally assessed as a process rather than merely a product, drawing on students' explanations, representations, noticing, generalisations, interviews, observations, video records, and written work. That matters because a correct final answer alone rarely reveals whether a student actually grasps the underlying relationships in a quadratic situation.

At the same time, the indicators remain fragmented across theoretical perspectives. One study may operationalise reasoning through covariation, another through generalisation, and another through intuitive-reflective processes. This fragmentation indicates a need for an integrated assessment framework capable of capturing several dimensions of mathematical reasoning within a coherent quadratic-learning trajectory.

Future Research Directions (RQ4)

Table 7 summarizes ten research priorities that follow from the synthesis, spanning sample diversity, longitudinal investigation, integration of reasoning dimensions, instructional task design, assessment, and technology-mediated learning. Two gaps stand out immediately. The included studies frequently relied on small or context-specific samples, pointing to a need for research with more diverse participants, educational settings, and learning contexts. Longitudinal work is also needed to track how students' mathematical reasoning develops across extended instructional sequences, rather than within isolated tasks or short-term episodes.

Table 7. Future Research Directions in Mathematical Reasoning in Quadratic Functions

No.	Evidence-Based Research Gap	Future Research Direction	Rationale from the Included Studies
1	**Limited sample sizes and context-specific studies**	Conduct studies with larger and more diverse samples across schools, regions, and educational contexts	Several studies used very small or highly specific samples, including two students, two student pairs, or a single community-based setting.
2	**Limited evidence of longitudinal development**	Examine how mathematical reasoning develops and is sustained across learning sequences and longer periods	Existing studies frequently examine reasoning within particular units, tasks, or cross-sectional settings, limiting conclusions about sustained development.
3	**Fragmentation of reasoning perspectives**	Develop integrated frameworks connecting covariational, representational, visual, algebraic, functional, generalising, justificatory, and reflective reasoning	The included studies examine different reasoning dimensions rather than a common integrated framework.
4	**Limited understanding of representation coordination**	Investigate how students progressively coordinate visual, graphical, symbolic, numerical, and contextual representations	Several included studies emphasize multiple representations and representation coordination in quadratic reasoning.
5	**Insufficient understanding of learner variability**	Examine differences in reasoning pathways according to prior knowledge, mathematical ability, and learning characteristics	The included studies indicate differences associated with prior knowledge and learner characteristics.
6	**Limited transfer from research-based teacher support to classroom practice**	Conduct intervention and implementation studies examining how teacher reasoning-support moves are sustained in authentic classrooms	Teacher-support research has demonstrated promising reasoning-support moves, but classroom transfer remains insufficiently established.
7	**Lack of an integrated assessment framework**	Develop and validate multimodal, process-oriented assessment frameworks for mathematical reasoning in quadratic learning	The studies use diverse evidence sources without a common assessment framework.
8	**Limited exploration of instructional task design**	Investigate how task characteristics, prompts, contexts, and sequences influence the emergence and development of mathematical reasoning	The included studies indicate that task structure and representation influence reasoning opportunities.
9	**Limited integration of contextual and culturally responsive learning**	Examine quadratic-function reasoning through diverse authentic, contextual, and culturally meaningful situations	Contextual and culturally responsive learning appeared in some studies but remained limited in scope.
10	**Limited technology-mediated evidence within the final core corpus**	Investigate how dynamic and interactive technologies can support representation, visualisation, exploration, and reasoning in quadratic-function learning	Dynamic mathematics/graphing technology (e.g., GeoGebra, Desmos) was not an explicit focus of the final 10-study core corpus; therefore, this is a future research opportunity rather than an established finding.

Note. These research directions are review-derived implications synthesized from the findings, limitations, and recommendations of the included studies rather than verbatim recommendations from individual studies.

Three cross-cutting patterns deserve comment beyond the individual rows in Table 7. First, the fragmentation of reasoning perspectives and the lack of an integrated assessment framework (Rows 3-5, 7) point to the same underlying gap: existing studies rarely connect covariational, representational, generalising, and justificatory reasoning within one coherent

model, so future work should prioritize such integration over further isolated single-construct studies. Second, the task-design and contextual-learning priorities (Rows 8-9) suggest that how reasoning opportunities are structured matters as much as which representations are used. Third, the technology gap (Row 10) should be read strictly as an artifact of this review's corpus and search strategy, not as evidence that dynamic mathematics/graphing tools such as GeoGebra or Desmos are ineffective; it remains a genuinely open empirical question (see Limitations).

Integrated synthesis

The final corpus comprises only ten studies, several with very small and context-specific samples, including two based on two student pairs (Wilkie, 2022, 2024) and others drawing on single classrooms or community-based settings; the synthesis offered here is accordingly a provisional research agenda rather than a set of generalisable claims. Across the four research questions, a consistent pattern nonetheless emerges: students' challenges, the instructional features associated with reasoning opportunities, and the process-oriented assessment indicators identified in this review all point to the same underlying need not more isolated studies of quadratic functions, but research that deliberately connects challenges, instructional design, and assessment within systematically sequenced learning trajectories (see Conclusion).

Limitations

This review's findings should be read in light of several limitations. First, the search was restricted to a single database (Google Scholar); the exact calendar date of the search was not recorded, and an identical search conducted later cannot be guaranteed to reproduce the original 80-record count because Google Scholar's index is not static. Second, the search string combined only the core terms "mathematical reasoning"/"reasoning ability" with "quadratic function"/"quadratic equation" and did not include terms for specific reasoning sub-constructs (e.g., covariational reasoning, generalisation, justification); a broader, multi-database search using an expanded string is recommended for future replications. Third, journal eligibility was verified using whether a journal had ever been assigned a Scimago Journal Rank (SJR) quartile; this criterion provides a defensible minimum quality signal but is applied at the journal level rather than the article level.

The specific identities of the six full-text reports excluded on this basis were not retained in the extraction records, so a sensitivity analysis re-examining whether their inclusion would alter the review's main conclusions cannot be conducted retrospectively; this is acknowledged as a transparency gap. Likewise, the ten included studies' individual SJR quartile ranks (per

journal, per year) are not tabulated in this review, because verifying and dating each journal's current quartile requires live, per-journal lookups against the Scimago database that could not be reliably completed within the production of this manuscript. A renewed check prompted by peer review confirms this concretely for two of the ten included studies, Rahmawati & Putri (2022a) in AKSIOMA: Jurnal Program Studi Pendidikan Matematika and Puspitasari et al., (2026) in the Journal of Advanced Sciences and Mathematics Education, both indexed in Indonesia's Garuda and DOAJ directories but not confirmed as Scopus/Scimago-indexed at the time of this check. We have not removed these two studies, since the SJR-quartile rule was applied at screening as a minimum quality filter rather than as the sole basis for relevance, and removing them would also reduce the reviews already limited Indonesian representation; we flag this honestly as an unresolved tension rather than asserting indexing we could not verify, and defer to the editor whether these two studies should instead be excluded.

Future replications of this review should log each journal's verified quartile, the year it refers to, and the date it was checked, and should retain the identities of excluded reports specifically so that a sensitivity analysis can be conducted. Fourth, publication-period and language restrictions were applied during screening rather than as Google Scholar search-engine filters, so the initial 80-record count already reflects manual removal of four out-of-period and one non-English record rather than engine-level exclusion.

Fifth, study selection and quality appraisal were conducted by the first author with disagreements resolved through discussion with the second author, rather than through independent dual screening with a formal inter-rater reliability statistic; this is a common constraint of small-team reviews but limits the extent to which these decisions can be considered fully independent. Sixth, the corpus combines studies focused on quadratic equations (e.g., Baybayon & Lapinid, 2024; Reid O'Connor & Norton, 2024) with studies focused on quadratic functions; although the two domains are conceptually related and are named explicitly where a finding's primary source is one or the other (e.g., in Table 4's traceability note), this is not a systematic tag on every table cell, and some themes draw mainly on the equation-focused subset.

Seventh, several themes in the Results rest on only one or two studies and should be read as illustrative rather than established; in particular, Wilkie (2022) and Wilkie (2024) both report findings from two pairs of Year 10 students in Australia, and although they are published as separate articles in different journals, the possibility that they draw on the same or an

overlapping sample could not be ruled out from the published texts, so themes supported jointly by both reports may rest on fewer independent sources than the raw study count suggests.

Finally, the review did not identify studies that examined dynamic mathematics or graphing technology (e.g., GeoGebra, Desmos) as an explicit mechanism for supporting reasoning in quadratic functions; this reflects a gap in the specific corpus retrieved by this review's search strategy and should not be read as evidence that such technology is ineffective or unstudied in the broader field. A recent study outside this review's search corpus illustrates the point: Muhfyana et al. (2026) found that secondary students using Desmos to solve quadratic-function problems showed adequate procedural reasoning (e.g., substitution, linking algebraic and graphical features) but continued to struggle with conceptual understanding of graph characteristics, suggesting that technology-mediated reasoning in this domain remains an open and under-synthesized area for future reviews.

CONCLUSION

This systematic literature review synthesized 10 empirical studies addressing mathematical reasoning in quadratic functions. Students' challenges centred on conceptual understanding, prerequisite algebraic knowledge, representation coordination, covariational reasoning, generalisation, and justification (RQ1). Instructional approaches associated with opportunities for reasoning combined multiple representations, figural-pattern generalisation, contextual and collaborative learning, teacher mediation, and formative feedback, although no single approach dominated the corpus (RQ2). Assessment practices increasingly treated reasoning as an observable process representing, noticing, generalising, justifying, and reflecting rather than only a final answer, although indicators remained fragmented across studies (RQ3). Future-research priorities identified across the corpus include larger and more diverse samples, longitudinal designs, integrated reasoning frameworks, systematically sequenced instructional tasks, process-oriented assessment, and technology-mediated learning environments (RQ4).

The contribution of this review lies in connecting these four dimensions challenges, instruction, assessment, and future directions within a single synthesis, indicating that the central need in this literature is not simply additional studies of quadratic functions but more integrated research linking students' reasoning difficulties, instructional design, and assessment. Theoretically, the review situates domain-specific reasoning in quadratic functions within broader conceptualisations of mathematical reasoning; practically, it offers a provisional

map that can inform curriculum design, instructional development, and assessment construction.

Given the small, single-database, heterogeneous evidence base detailed in the Limitations, these conclusions should be read as a provisional synthesis and research agenda rather than as generalisable claims. Future work should prioritize systematically sequenced learning trajectories that connect contextual exploration, multiple representations, and formalisation, evaluated with larger and more diverse samples and multi-database searches; dynamic mathematics/graphing technology (e.g., GeoGebra, Desmos) represents one such promising but, within this specific corpus, as yet empirically untested direction.

REFERENCES

Abdelmalak, M. M. M. (2025). Developing prospective teachers' skills in supporting student reasoning on quadratic functions. *Research in Mathematics Education*, 1–27. https://doi.org/10.1080/14794802.2025.2540788

Altindis, N., & Fonger, N. L. (2025). A Culturally Inclusive Mathematics Learning Environment Framework: Supporting Students' Representational Fluency and Covariational Reasoning. *Education Sciences*, *15*(8), 1–23. https://doi.org/10.3390/educsci15080980

Bakker, A., Cai, J., & Zenger, L. (2023). Future themes of mathematics education research: an international survey before and during the pandemic. *Educacion Matematica*, *35*(2), 9–46. https://doi.org/10.24844/EM3502.01

Baybayon, G. G., & Lapinid, M. R. C. (2024). Students' common errors in quadratic equations: Towards improved mathematics performance. *Infinity Journal*, *13*(1), 83–98. https://doi.org/10.22460/infinity.v13i1.p83-98

Berge, R. L., & Solberg, E. M. (2026). Assessing mathematical reasoning: analyzing imitative and creative reasoning by students. *Educational Studies in Mathematics*, *122*(3), 407–425. https://doi.org/10.1007/s10649-026-10487-4

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2024). Trends in mathematics education and insights from a meta-review and bibliometric analysis of review studies. *ZDM - Mathematics Education*, *56*(2), 165–188. https://doi.org/10.1007/s11858-024-01587-7

Engelbrecht, J., & Borba, M. C. (2024). Recent developments in using digital technology in mathematics education. *ZDM - Mathematics Education*, *56*(2), 281–292. https://doi.org/10.1007/s11858-023-01530-2

Gustafsson, M. (2026). Empirical findings on the teaching and learning of quadratic equations: a systematic review. *Educational Studies in Mathematics*, *123*(1), 73–118. https://doi.org/10.1007/s10649-026-10493-6

Hohensee, C., Gartland, S., Willoughby, L., & Melville, M. (2021). Backward transfer influences from quadratic functions instruction on students' prior ways of covariational reasoning about linear functions. *The Journal of Mathematical Behavior*, *61*(Drl 1651571), 100834. https://doi.org/10.1016/j.jmathb.2020.100834

Meriza, D., Hiltrimartin, C., Hartono, Y., & Indaryanti, I. (2022). Student Activity Sheet Development Quadratic Equations and Functions based on Problem Solving in Junior High School. *Advances in Social Science, Education and Humanities Research*, *656*(NaCoME 2021), 79–89.

Muhfyana, A., Pasaribu, C. R., Sancia, D. M., Girsang, D. O. M., Shakila, M. K., Sianipar, T. S., & Situmorang, R. N. (2026). Analysis of Students' Mathematical Reasoning Ability in Quadratic Functions through the Use of Desmos. *Jurnal Riset Rumpun Matematika Dan Ilmu Pengetahuan Alam*, *5*(1), 201–210. https://doi.org/10.55606/jurrimipa.v5i1.8974

OECD. (2023). *The State of Learning and Equity in Education: Vol. I*. https://doi.org/https://doi.org/10.1787/53f23881-en. IS

OECD. (2026). *PISA 2025 Results (Volume I): Vol. I* (Issue Volume I). https://doi.org/https://doi.org/10.1787/73451bc5-en

Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., Brennan, S. E., Chou, R., Glanville, J., Grimshaw, J. M., Hróbjartsson, A., Lalu, M. M., Li, T., Loder, E. W., Mayo-Wilson, E., McDonald, S., … Moher, D. (2021). The PRISMA 2020 statement: An updated guideline for reporting systematic reviews. *Bmj*, *372*. https://doi.org/10.1136/bmj.n71

Puspitasari, E. T. P., Subanji, Irawati, S., Anwar, L., & Darmawan, P. (2026). Intuitive and reflective mathematical reasoning in solving quadratic function problems: A dual-process perspective. *Journal of Advanced Sciences and Mathematics Education*, *6*(2), 456–476. https://doi.org/10.58524/jasme.v6i2.1237

Putri, D. S., Hiltrimartin, C., Hartono, Y., & Aisyah, N. (2024). *Pengembangan LKPD Fungsi Kuadrat Berbasis Pembuktian Dengan Dengan Model Argumentasi Toulmin Untuk Memfasilitasi Kemampuan Argumen Matematis*. *13*(4), 1157–1173. https://doi.org/https://doi.org/10.24127/ajpm.v13i4.9181

Rahmawati, S., & Putri, R. I. I. (2022). Penalaran Matematis Siswa Dalam Pembelajaran Fungsi Kuadrat Menggunakan PMRI Dan Collaborative Learning Berbantu Media Video. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *11*(1), 577–588. https://doi.org/10.24127/ajpm.v11i1.4622

Ramírez, R., Cañadas, M. C., & Damián, A. (2022). Structures and representations used by 6th graders when working with quadratic functions. *ZDM - Mathematics Education*, *54*(6), 1393–1406. https://doi.org/10.1007/s11858-022-01423-w

Reid O'Connor, B., & Norton, S. (2024). Exploring the challenges of learning quadratic equations and reflecting upon curriculum structure and implementation. *Mathematics Education Research Journal*, *36*(1), 151–176. https://doi.org/10.1007/s13394-022-00434-w

Säfström, A. I., Lithner, J., Palm, T., Palmberg, B., Sidenvall, J., Andersson, C., Boström, E., & Granberg, C. (2024). Developing a diagnostic framework for primary and secondary students' reasoning difficulties during mathematical problem solving. *Educational Studies in Mathematics*, *115*(2), 125–149. https://doi.org/10.1007/s10649-023-10278-1

Salam, M., Hasnawati, H., Andini, I. A. P., Suhar, S., & Lambertus, L. (2023). Analisis Kemampuan Penalaran Matematis Siswa Ditinjau Dari Kemampuan Awal. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *12*(2), 2351–2362. https://doi.org/10.24127/ajpm.v12i2.7448

Salem, A. H., Azzam, S. M., Emam, O. E., & Abohany, A. A. (2024). Advancing cybersecurity: a comprehensive review of AI-driven detection techniques. *Journal of Big Data*, *11*(1), 105. https://doi.org/10.1186/s40537-024-00957-y

Santos-Trigo, M. (2024). Problem solving in mathematics education: tracing its foundations and current research-practice trends. *ZDM - Mathematics Education*, *56*(2), 211–222. https://doi.org/10.1007/s11858-024-01578-8

Weigand, H. G., Trgalova, J., & Tabach, M. (2024). Mathematics teaching, learning, and assessment in the digital age. *ZDM - Mathematics Education*, *56*(4), 525–541. https://doi.org/10.1007/s11858-024-01612-9

Widjaja, W., Vale, C., Herbert, S., Loong, E. Y. K., & Bragg, L. A. (2021). Linking comparing and contrasting, generalising and justifying: a case study of primary students' levels of justifying. *Mathematics Education Research Journal*, *33*(2), 321–343. https://doi.org/10.1007/s13394-019-00306-w

Wilkie, K. J. (2022). Generalization of quadratic figural patterns: Shifts in student noticing. *The Journal of Mathematical Behavior*, *65*, 100917. https://doi.org/10.1016/j.jmathb.2021.100917

Wilkie, K. J. (2024). Coordinating visual and algebraic reasoning with quadratic functions. *Mathematics Education Research Journal*, *36*(1), 33–69. https://doi.org/10.1007/s13394-022-00426-w