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From representation to strategy: reconstructing an analytical model of students' cognitive strategies in open-ended geometry problem solving

Anis Farida Jamil[1)*], Buaddin Hasan[2)], Arif Hidayatul Khusna[3)]
[1) 3)] Universitas Muhammadiyah Malang, Malang, Indonesia
[2)] STKIP PGRI Bangkalan, Jawa Timur, Indonesia
*anisfarida@umm.ac.id

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Abstract

Open-ended geometry problems allow students to generate diverse mathematical representations and solution strategies. However, previous research has emphasized solution products rather than the strategies reconstructed from students' mathematical work. This study aims to identify students' mathematical representations and solution strategies and develop a cognitive strategy model for solving open-ended geometry problems. Using a theory-informed qualitative approach, 33 junior high school students completed a task requiring them to divide an equilateral triangle into four equal-area regions using multiple strategies. Initial analysis revealed that 24 students generated a single strategy, while nine produced multiple strategies. Two students with contrasting, information-rich solutions were purposively selected for in-depth analysis. Their written artifacts were analyzed from representation and problem-solving perspectives, through cross-case comparison, and through conceptual reconstruction. The findings revealed two distinct patterns: one student generated alternative strategies primarily through visual representations, while the other developed strategies by applying geometric principles. Both patterns suggested iterative relationships between representations and strategy development. These findings informed a preliminary cognitive strategy model comprising four interacting components: problem representation, representation transformation, strategy development, and strategy verification. The model conceptualizes strategy development as an iterative coordination between representations and strategies rather than a strictly linear process. As the model was reconstructed from two students' written artifacts, it represents a theory-informed analytical interpretation rather than a direct observation of cognitive processes.

Keywords: Geometry; Mathematical Representation; Qualitative Case Study; Strategy Development; Written Problem-Solving

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INTRODUCTION

Mathematics learning in the 21st century is no longer solely focused on correct answers. Contemporary mathematics education policies and curricula increasingly emphasize the development of Higher-Order Thinking Skills (HOTS) to prepare students for complex challenges (NCTM, 2020; Nissa, 2022; OECD, 2015). While HOTS is frequently operationalized through the upper levels of the revised Bloom's Taxonomy–specifically

analyzing, evaluating, and creating–it is a multifaceted construct. Beyond this taxonomy, a complete account of HOTS also encompasses broader competencies such as non-routine problem solving, critical reasoning, and creative thinking (Tamaulina, 2025; Widana, 2018) . Therefore, the Indonesian government is striving to continuously develop curricula at both elementary and secondary levels to support students' HOTS (Tanujaya et al., 2021). In the Revised Bloom's Taxonomy, HOTS is at the Analyze (C4), Evaluate (C5), and Create (C6) levels (Wilson, 2016). HOTS is not a single skill but rather a complex thinking skill involving higher-level cognitive processes. This means that when students work on math problems, they don't just memorize formulas (C1) or apply procedures (C3), but they also have to analyze the problem's conditions, evaluate strategies, and create various alternative solutions.

The development of HOTS in mathematics learning is realized through the ability to solve problems, reason, and think creatively (Leonisa & Soebagyo, 2022; Seman et al., 2017; Wulandari & Machromah, 2024). In the context of problem solving, creative thinking is not only demonstrated by the ability to generate many solutions or ideas (fluency), but also the ability to use different solution strategies (flexibility) and produce new solutions (originality) (Silver, 1997). Therefore, mathematical problems that provide the opportunity for more than one strategy or solution to emerge are the right context for exploring students' creative thinking.

One problem characteristic that can facilitate the emergence of these indicators of creative thinking is open-ended geometry problems. Open-ended geometry problems provide students with the opportunity to generate more than one strategy or more than one solution that are both mathematically correct. Previous research has shown that students' creative thinking can be triggered by open-ended problems in geometry (Pradiarti et al., 2024). This is because open-ended geometry problems can encourage the development of students' mathematical creativity through the emergence of various representations, strategies, and solutions in solving problems (Nu'man & Azka, 2023; Pradiarti et al., 2024).

The diversity of strategies and solutions emerging in open-ended geometry problems not only demonstrates students' ability to solve problems in more than one way but also demonstrates how they construct an understanding of the geometric concepts used. When students are faced with problems with multiple possible solutions, they need to connect previously acquired concepts to determine the most appropriate strategy (Jamil, Siswono, & Setianingsih, 2023a, 2023b; Jamil, Siswono, Setianingsih, et al., 2023). Therefore, problem-solving strategies cannot be viewed merely as procedures for finding solutions, but as a manifestation of the cognitive processes that occur in problem-solving (Aragón et al., 2024;

Turner, 2011). Students producing multiple-solution tasks require adequate prior knowledge as cognitive resources, so that a diversity of solutions does not emerge without a strong knowledge base (Mahlaba, 2020). Thus, in-depth exploration of students' cognitive processes in generating diverse solutions or resolutions is important.

To clarify the conceptual boundaries within this study, it is essential to distinguish between creativity, the generation of multiple solutions, mathematical representations, and cognitive strategy formation. Mathematical creativity is often operationalized through the production of multiple solutions, which are typically evaluated using divergent thinking metrics: fluency (the number of correct solutions), flexibility (the variety of mathematical approaches), and originality (the uniqueness of the solutions) (Alabbasi et al., 2025; Paz-Baruch et al., 2025). While fluency, flexibility, and originality effectively assess the outcomes or products of creative thinking, they do not fully capture the underlying cognitive mechanisms. Representations serve as the tools or externalized mental models' students use to express their mathematical understanding. In contrast, cognitive strategy formation refers to the dynamic, step-by-step mental pathways and decision-making processes students employ to manipulate these representations to solve a problem. Thus, while fluency, flexibility, and originality measure the creative product, strategy formation explains the internal cognitive processes that drive the generation of these multiple solutions.

Although research on open-ended problems has been extensive, most research still focuses on student-defined outcomes, such as the number of solutions generated, the level of creativity, the type of representation, or the characteristics of the strategies used (Matondang et al., 2023; Pratiwi & Ekawati, 2019; Puspasari, 2024). This research provides a picture of the quality of students' solutions, but it fails to explain how a strategy is formed during the problem-solving process. Strategies evident in students' written solutions can be viewed as traces of the mathematical activity that occurred during problem-solving. Modifications to diagrams, the use of symbols, the addition of constructions, the emergence of alternative solutions, and written justifications provide evidence that can be used to trace the relationship between representations and strategies. Although written evidence does not offer direct access to the entirety of a student's mental processes, patterns observed in these artifacts allow for an analytical reconstruction of how solution strategies evolved. Thus, the study of written solutions does not aim to make direct claims about the cognitive processes at play, but rather to reconstruct cognitive-strategic relationships supported by observable mathematical evidence.

Research on mathematical creativity still focuses more on the products of thinking than on the process of idea formation (Schindler & Lilienthal, 2020). Similar research findings also show that analysis of the problem-solving process remains a challenge as well as an opportunity for research in the field of mathematics education (Jiang et al., 2021; Schoevers et al., 2020). However, there is research that has examined the process of solving open-ended geometry problems, but its focus differs from this study. Previous research analyzed collaborative geometry problem-solving activities by exploring the writing process, proof, and evaluation in group work, thus placing more emphasis on the learning experience and the process of solving problems in groups (Borgersen, 2004). Other research has developed a model of the mathematical creativity process that includes reading and exploring problems, changing perceptions, searching for ideas, incubating, implementing ideas, verifying, and self-regulating (Simamora & Kamara, 2024). Meanwhile, other research describes differences in students' creative thinking abilities based on fluency, flexibility, and novelty and groups students into high, medium, and low levels of creativity based on the variety of strategies and solutions they produce (Andrianti et al., 2026). Based on a review of recent literature–specifically targeting peer-reviewed articles focused on mathematical creativity and cognitive processes in geometry education over the last decade–the authors found limited research that specifically reconstructs a cognitive strategy model from students' written artifacts in open-ended geometry problem solving.

This study aims to develop an analytical mode of students' cognitive strategies in solving open-ended geometry problems as a framework that explains the mechanisms of strategy formation and development based on students' written solutions. Based on the explanations above, this study raises two problem formulations as follows:

1. What are the characteristics of students' mathematical representations and problem-solving strategies in solving open-ended geometry problems?
2. How can an analytical model of cognitive strategies be reconstructed from students' written solutions in open-ended geometry problem-solving?

METHOD

Research Approach and Type

This research is qualitative research oriented towards conceptual model development. This research aims to develop a conceptual model through a synthesis of established theory with empirical evidence obtained from students' written solutions. The analysis was conducted

using a theory-informed deductive-inductive approach, in which the Open-Ended Approach theory, mathematical representation theory, and problem-solving theory were used as the initial framework in the analysis process, while still allowing for the emergence of new categories and relationships derived from empirical data. The results of the analysis were then integrated and synthesized into a Cognitive Model of Students' Strategies in Solving Open-Ended Geometry Problems (Collins & Stockton, 2018; Fife & Gossner, 2024; Jabareen, 2009; Naeem et al., 2023). In this study, students' written solutions were treated as the primary empirical evidence for analyzing mathematical representation and problem-solving strategy. The analysis did not assume direct access to students' internal mental processes. Instead, observable written traces, including diagrams, auxiliary lines, symbolic notations, changes in geometric configurations, variations of strategies, and written verification, were used as the basis for analytical reconstruction.

Research Subjects

The subjects of this research were 33 eighth-grade students. The detailed demographic information regarding the male and female students is presented in Table 3. These 33 students were recruited from a single intact class at a public junior high school, which represents a typical urban educational context. All written solutions were then analyzed initially as the basis for selecting the research subjects. Subject selection was performed using purposive sampling, selecting participants who were able to provide information-rich cases in accordance with the research objectives (Patton, 2014). The selection and analysis of the students' written solutions were based on four specific criteria, which were assessed as follows:1) Mathematical validity; 2) Clarity; 3) Completeness; and 4) Contrast. To ensure the reliability and objectivity of the analysis, these criteria were assessed independently by two researchers.

Based on the selection results, two students were chosen as the main subjects of the study because they met all the criteria and showed contrasting characteristics of problem-solving strategies. The selection of these two subjects aimed to gain a deeper understanding of strategy formation in solving open-ended geometry problems, not to make statistical generalizations, as is characteristic of qualitative research which emphasizes the depth and richness of information over the number of participants (Creswell, 2016; Patton, 2014).

Research Instruments

To maintain consistency of analysis, researchers used supporting instruments in the form of open-ended geometry assignments and analysis guidelines developed based on the Open-

Ended Approach theory (Munroe, 2015), Theory of Semiotic Representation (Duval, 2006), and Mathematical Problem-Solving (Schoenfeld, 1985).

These theories were operationalized according to their functions in the analysis. The Open-Ended Approach guided the design of a task that allowed multiple mathematically valid solution strategies. Duval's theory guided the identification of students' visual, symbolic, and verbal representations, including transformations between representations. Schoenfeld's perspective guided the analysis of how students selected, developed, modified, and verified their solution strategies. The relationship among these theoretical perspectives and the analysis is summarized in Table 1.

Table 1. Operationalization and Construct-to-Data Mapping in the Analysis of Students' Solution Strategies

Theoretical Perspective	Operational Focus	Evidence Identified in Students' work	Application in Coding Process
Open-Ended Approach	Multiple valid solution strategies	Different solution path used for the same problem	Coded to classify the diversity and flexibility of the solutions, establishing the baseline for identifying multiple distinct strategy patterns.
Duval's Semiotic Representation	Representation and transformation	Diagram, auxiliary lines, symbols, equations, and verbal explanations	Formed the coding categories for Problem Representation (initial diagramming) and Representation Transformation (modifying diagrams, e.g., partitioning sides).
Schoenfeld's Problem Solving	Strategy development and verification	Selection, modification, sequence, and checking of strategies	Formed the coding categories for Strategy Development (exploring configurations/concepts) and Strategy Verification (written area arguments and mathematical proofs).

The first supporting instrument was a single, open-ended geometry task designed to provide students with the opportunity to generate more than one strategy or mathematically correct solution. Since students were asked to generate as many solutions as possible, one task could provide multiple strategy patterns for analysis. Prior to use, the task was validated by two mathematics education lecturers and one high school mathematics teacher to ensure appropriate content, clarity of language, and its ability to generate multiple solution strategies.

The task given to students are as follows.

Given an equilateral triangle ABC as shown in the Figure 1, divide the triangle into 4 equal parts, defined as regions of equal area. In your constructions, you may use straight line segments, mark points on the sides, or draw any auxiliary objects. Create as many solutions as possible, along with explanations and reasons!

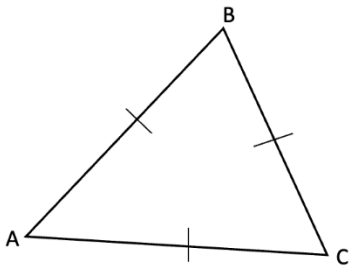


Figure 1. Equilateral Triangle ABC

Data Collection & Analysis Technique

Data collection in this study was conducted by administering an open-ended geometry assignment to 33 participants who had studied the area of triangles. Prior to the study, the researcher explained the research objectives, implementation procedures, and how to complete the assignment. The analysis was conducted in five stages. In the first stage, all students' written solutions were read in their entirety (data familiarization) to gain an understanding of the general characteristics of the representations and solution strategies. In this stage, the researcher identified the images, symbols, notations, guidelines, and sequence of solution steps used by each student. In the second stage, the coding process for the mathematical representations and solution strategies was carried out based on initial categories developed from the theory Duval (2006) and Schoenfeld (1985). Coding is carried out openly so that new categories that emerge from the data can still be accommodated if they do not match the initial categories.

To illustrate the coding process, Figure 2 demonstrates how one piece of student work was analyzed. The initial drawing externalizing the problem was coded as Problem Representation (PR). Subsequent geometric modifications, such as shifting from an internal quadrilateral to lines drawn from vertices to the opposite sides, were coded as Representation Transformation (RT). The student’s written justifications supporting the equal-area claim were coded as Strategy Verification (SV), and the generation of multiple distinct constructions was coded as Strategy Development (SD).

In the third stage, the coding results were compared using the constant comparative analysis technique to identify similarities, differences, and relationships between the strategies demonstrated by the two research subjects (Corbin & Strauss, 2015).

In the fourth stage, the patterns obtained are synthesized into conceptual relationships between components to obtain an initial design of the Student Cognitive Strategy Model. At this stage, a conceptualization process is carried out by integrating the results of empirical analysis and theoretical foundations to explain how mathematical representations contribute to the formation of problem-solving strategies in open-ended geometry problems (Jabareen, 2009; Naeem et al., 2023).

In the fifth stage, the resulting conceptual model was validated through expert review by two experts. Validators were asked to provide input on the clarity of the constructs, the relationships between components, the model's suitability to empirical data, and its consistency with the theory used. The validation results were used as a basis for model refinement until the final Cognitive Model of Students' Strategies in Solving Open-Ended Geometry Problems was obtained.

The preliminary Cognitive Strategy Model was reviewed by two experts in mathematics education with expertise in mathematical cognition and mathematical problem solving. The expert review focused on the conceptual clarity of the model components, their consistency with the empirical findings, and the coherence of the relationships represented in the model. These revisions resulted in the final Cognitive Strategy Model presented in Table 2.

Table 2. Expert Validation Feedback and Corresponding Revisions to the Cognitive Strategy Model

Pre-revision Model	Main Feedback from Validators	Revision to the Model
The initial model incorporated four key components: problem representation, representation transformation, strategy development, and strategy verification, but the relationships between them emphasized a sequential flow, potentially leading to the interpretation of the process as linear.	The relationships among the four components need to be clarified to demonstrate that strategy formation does not always proceed linearly; rather, it allows for reciprocal relationships and iterative processes involving representation, strategy development, and verification.	The final model retains the four key components while clarifying the directional relationships and feedback loops among them. The interaction between representation and strategy is more explicitly positioned as an iterative process driven by cognitive coordination, such that verification can redirect the transformation of representations or the subsequent development of strategies.
The initial model focuses on components and the relationships between representations and strategies, but the cognitive dimensions	The cognitive dimension of each component needs to be clarified to demonstrate the analytical function of each, while simultaneously limiting the interpretation, given that	The final model incorporates a cognitive focus for each component, while positioning cognitive coordination at the model's center. These dimensions are framed as

Pre-revision Model	Main Feedback from Validators	Revision to the Model
underlying each component are not yet explicitly depicted in the model's visualization.	the study's primary data source consists of students' written solutions.	theoretical interpretations reconstructed from written evidence, rather than as directly observed student mental processes.

RESULTS AND DISCUSSION

Selection of Research Subjects

The study began with an open-ended geometry assignment given to 33 eleventh-grade students at a high school in Malang City who had studied the area of triangles. All students completed the assignment individually, resulting in 33 written solutions as the initial data for the study. These solutions were then analyzed to identify the characteristics of the students' mathematical representations and solution strategies.

The initial analysis showed that 24 students solved the problems using a single solution strategy, while 9 students were able to generate more than one mathematically valid strategy. These findings indicate that most students still tend to use a single approach in solving open-ended geometry problems, while only a small number of students are able to explore various alternative solution strategies.

Furthermore, the subject selection process was carried out using a purposive sampling technique on nine students who produced more than one solution strategy. Subject selection was based on four criteria, namely: (1) producing more than one strategy or solution that is mathematically valid, (2) showing a clear mathematical representation through pictures, guide lines, symbols, or geometric notation, (3) writing down the solution steps completely so that it is possible to trace the relationship between the representation and the strategy, and (4) showing different strategy characteristics so that they provide rich information (information-rich cases) in accordance with the research objectives (Patton, 2015).

Among the nine students, two were selected because their written work provided the richest and most contrasting evidence for comparative analysis. One student displayed substantial variation through changes in visual configurations across solutions, while the other showed a more systematic reuse and generalization of geometric principles across different configurations. These contrasting written patterns were considered analytically important because they provided two complementary pathways for reconstructing how strategies developed from the observable artifacts. Subsequent in-depth analysis characterized these

pathways as representation-driven and concept-driven strategy development. The remaining seven students also produced multiple mathematically valid strategies; however, their written solutions showed either relatively similar representational patterns across strategies, less complete written evidence for tracing changes between representation and strategy, or less pronounced contrasts in strategy development. Therefore, although their solutions were relevant to the initial screening, they provided less analytically rich evidence for the comparative reconstruction targeted in this study.

Table 3. Summary of Multiple Strategies Generated by Students and Case Selection

Student	Number of Solutions	Relevant Evidence	Selection Status & Justification for In-Depth Analysis
Subject 1	4	Modified geometric visual configurations, added auxiliary lines, and partitioned sides.	**Selected (Focal Case 1)**: Provided the richest evidence with substantial variation through changes in visual configurations across solutions (representing the **representation-driven** pathway).
Subject 2	4	Systematically used midpoints and congruence symbols based on base-height relationships.	**Selected (Focal Case 2)**: Provided the richest contrasting evidence showing a systematic reuse and generalization of geometric principles across configurations (representing the **concept-driven** pathway).
Subject 3	3	Partitioned the base into four equal segments and connected them to the top vertex; used basic area formula of triangle to prove equality.	**Not Selected:** Produced valid strategies but showed relatively similar representational patterns across strategies.
Subject 4	2	Divided the triangle using one median, then subdivided the two resulting triangles using horizontal segments; attempted algebraic notation.	**Not Selected:** Produced valid strategies but provided less complete written evidence for tracing changes between representation and strategy.
Subject 5	3	Rotated the triangle to identify different bases; constructed combinations of smaller internal triangles using midpoints on two sides.	**Not Selected:** Produced valid strategies but showed less pronounced contrasts in strategy development compared to the focal subjects.
Subject 6	2	Constructed smaller triangles inside the main triangle using partial	**Not Selected:** Produced valid strategies but provided less complete written evidence for

		parallel lines; relied heavily on visual estimation without formulas.	tracing changes between representation and strategy.
Subject 7	3	Drew perpendicular bisectors from each side; attempted folding strategies represented through dashed lines connecting midpoints.	**Not Selected:** Produced valid strategies but showed relatively similar representational patterns across strategies.
Subject 8	4	Partitioned two sides equally and connected the corresponding points; combined lines from vertices with internal quadrilaterals.	**Not Selected:** Produced valid strategies but showed less pronounced contrasts in strategy development compared to the focal subjects.
Subject 9	2	Used a coordinate geometry approach on a drawn Cartesian plane; calculated areas using vertices coordinates.	**Not Selected:** Produced valid strategies but provided less complete written evidence for tracing changes between representation and strategy.

The selection of two subjects was not intended to represent the entire population, but rather to gain a deeper understanding of the variation in strategy formation in solving open-ended geometry problems. Both subjects showed different patterns of mathematical representation and solution strategies, thus allowing for a more comprehensive comparative analysis and reconstruction of the Student Cognitive Strategy Model. The research subject selection process is presented in Table 4.

Table 4. Results of Initial Selection of Research Participants

Selection Stage	Number of Students
Participants working on open-ended geometry assignments	33
Produce a solution strategy	24
Generate more than one solution strategy	9
Meets all research criteria	2 (Subject 1 & Subject 2)

Characteristics of Mathematical Representation and Students' Problem-Solving Strategies

Following the research subject selection process, an in-depth analysis of the written solutions of two selected students was conducted to identify the characteristics of mathematical representation and solution strategies used in solving open-ended geometry problems. The analysis was conducted based on two main aspects: mathematical representation (guided by Duval's Theory of Semiotic Representation) and problem-solving strategies (guided by

Schoenfeld's mathematical problem-solving). Mathematical representations are analyzed through images, symbols, notations, and geometric constructions used by students, while problem-solving strategies are analyzed through the sequence of steps, variations in approaches, and the relationship between representations and strategies developed during the problem-solving process.

Subject 1

Figure 2 illustrates the written solution of Subject 1, who successfully generated four different solution strategies for dividing a triangle into four regions of equal area. All strategies are accompanied by visual representations and brief explanations of the mathematical reasoning used. The written solution demonstrates that the subject generated several valid alternative solutions. The sequence and variation of constructions displayed indicate an exploration of strategies that can be reconstructed from the written artifact.

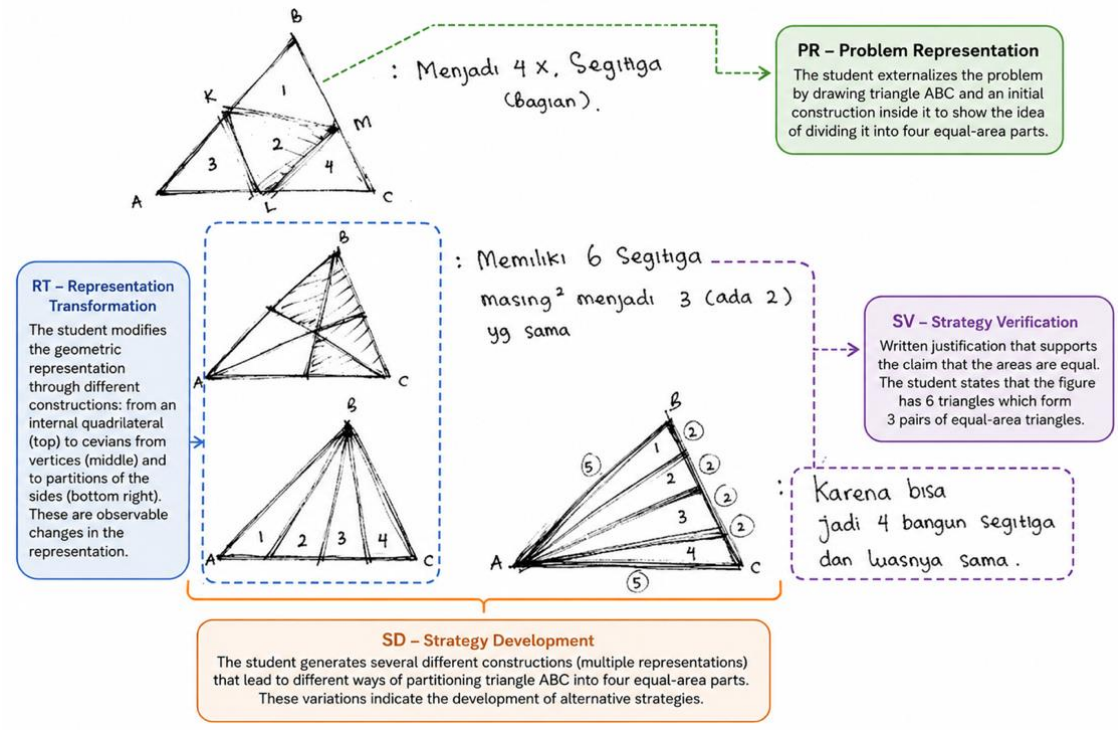


Figure 2. Results of Subject 1's Written Work

From the aspect of mathematical representation, Subject 1 consistently uses visual representation as the primary medium in developing solution ideas. Each strategy begins by redrawing the triangle, then adding auxiliary lines, points on the triangle's sides, and different line segment divisions. The representations constructed not only serve to depict geometric objects but also serve as a means to explore the relationships between the triangle's parts to

obtain four regions with equal area. The changes in representation in each strategy indicate that the subject gradually transforms the image according to the solution's needs.

In terms of solution strategies, Subject 1 demonstrated a variety of approaches to dividing a triangle into four equal parts. The initial strategy emphasized the formation of regions through the construction of lines within the triangle, while subsequent strategies developed by utilizing the division of the triangle's sides into several equal parts, then connecting them to certain corner points. In some strategies, the subject also provided a reason that the area of each region remained the same because they had the same base or height. This indicates that the strategies used were not procedural in nature, but were built on the utilization of relevant geometric properties.

Overall, Subject 1's solution demonstrated a close relationship between mathematical representation and solution strategy. In Subject 1's written response, each new strategy appears to be accompanied by a change or modification to the previously constructed representation, followed by adjustments to the geometric construction until the desired solution was obtained. This finding indicates that mathematical representation serves as the basis for the formation and development of solution strategies for open-ended geometric problems, so that the two aspects cannot be separated in the problem-solving process.

Subject 2

Subject 2's written work is presented in Figure 3. Similar to Subject 1, this subject formulated four different strategies, but arrive at them through a highly concept-driven pathway. Unlike Subject 1, who explored various geometric constructions, Subject 2 developed a solution based on a single basic idea that was then modified into several alternatives. Each strategy was accompanied by a visual representation and a mathematical explanation that showed why the regions formed had equal areas.

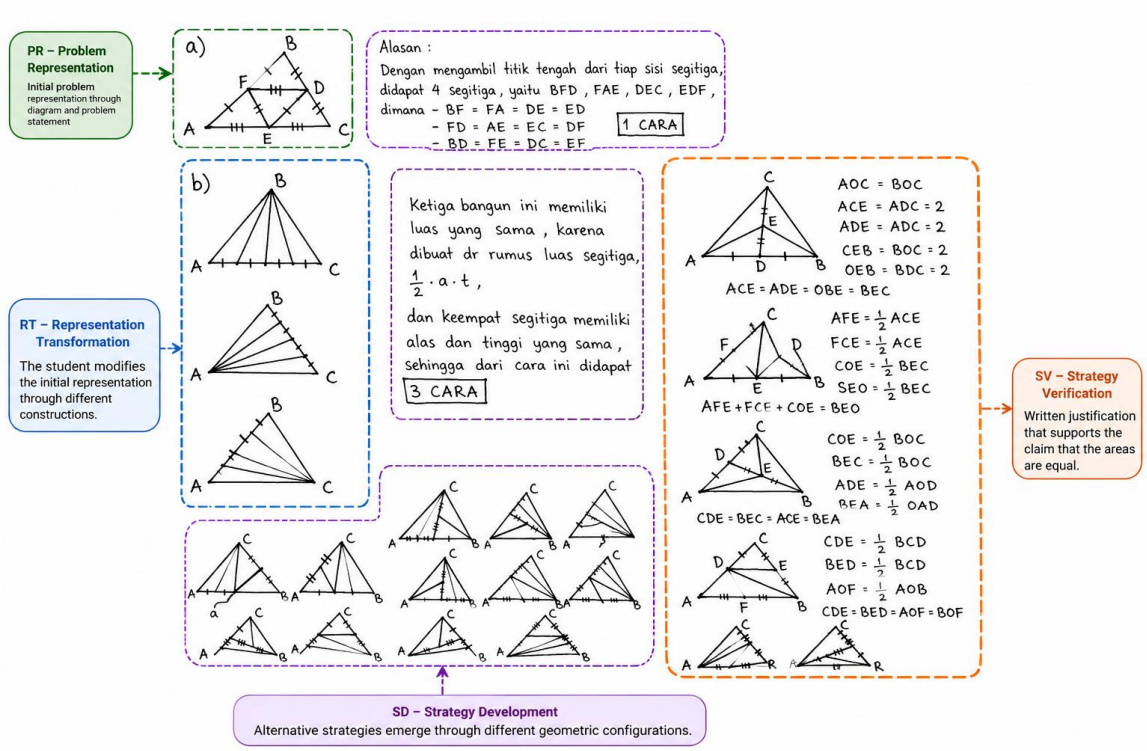


Figure 3. Results of Subject 2's Written Work

In terms of mathematical representation, Subject 2 demonstrated a more systematic and structured representation. Most strategies were constructed using midpoints, dividing sides into equal parts, and relationships between the bases and heights of triangles. In addition to using images, Subject 2 also added symbols representing equal lengths of line segments and relationships between areas of triangles to support their arguments. These representations demonstrate that students use images not only as illustrations but also as a means to organize the geometric relationships underlying each solution strategy.

In terms of solution strategies, Subject 2 demonstrated a solution pattern that developed through the generalization of a single geometric principle into several different configurations. The first strategy used the midpoints on each side of a triangle to form four triangles of equal area. Subsequent strategies developed by applying the same principle to different sides or points, and utilizing the concept that triangles with the same base or height will have the same area. In several solutions, Subject 2 consistently cited the relationship between triangle areas, midpoints, and segment congruence as the basis for the written justification regarding the equality of the areas of the formed regions.

Overall, Subject 2's solutions demonstrate that mathematical representations and solution strategies develop in an integrated manner. The representations constructed do not undergo as many changes in form as in Subject 1, but are used consistently to develop a variety of strategies

based on the same geometric principle. The characteristics of the written artifact reveal a solution pattern that can be reconstructed as the application and generalization of geometric principles across different representations. This generalization was inferred directly from the student's written work, as the student did not merely modify visual shapes but provided mathematical proofs based on area relations to justify their strategies. Specifically, the student explicitly cited the fundamental triangle area formula ($Area = \frac{1}{2} \times base \times height$) and established that triangles sharing the same height and having equal base segments will inherently have equal areas. Generalization was evident because the student systematically applied this exact core mathematical principle across multiple distinct geometric configurations–such as dividing the base into equal segments or using midpoints (e.g., $BF = FA = DE = ED$)–to mathematically generate and verify all alternative strategies. Thus, the relationship between mathematical representations and solution strategies in Subject 2 demonstrates a more conceptual pattern, where a single geometric principle is used as a basis for generating several different solution strategies.

Comparison of Mathematical Representation and Problem-Solving Strategies for Subject 1 and Subject 2

The analysis results show that both subjects were equally capable of generating more than one valid solution strategy in solving open-ended geometry problems, but exhibited different characteristics of mathematical representation and solution strategies. Subject 1 tended to develop strategies through visual representation transformation, namely by modifying images, adding guidelines, and changing geometric constructions to obtain different solution alternatives. In contrast, Subject 2 showed a tendency to develop various strategies through generalization of geometric principles, such as the use of midpoints, equality of line segment lengths, and the relationship between the areas of triangles, resulting in relatively fewer changes to the representation. Nevertheless, both subjects showed that mathematical representation plays a role as a basis for developing solution strategies. This finding indicates that the relationship between mathematical representation and solution strategies is not linear, but develops through different patterns in each student. These differences in characteristics indicate variations in the mechanisms in the formation of open-ended geometry problem-solving strategies, which then serve as the basis for reconstructing the Student Cognitive Strategy Model. Table 5 shows a comparison of the characteristics of mathematical representation and problem-solving strategies between subjects.

Table 5. Comparison of Mathematical Representation Characteristics and Problem-Solving Strategies of Subject 1 and Subject 2

Aspect	Subject 1	Subject 2
Mathematical representation	Representation develops through image transformation and the addition of auxiliary lines.	Relatively fixed representation, supported by geometric symbols and relationships
Strategy development	Explore various geometric constructions	Generalization of one geometric principle to various configurations
Basis of settlement	Modification of visual representation	Utilization of the concept of area similarity and geometric properties
Main characteristics	Representation-driven strategy	Concept-driven strategy

Reconstruction of Cognitive Model of Students' Strategies in Solving Open-Ended Geometry Problems

Based on the analysis of the written solutions of both subjects, it was found that mathematical representations and solution strategies have a mutually influential relationship during the process of solving open-ended geometry problems. Although both subjects produced more than one valid solution strategy, they showed different patterns of strategy development. Subject 1 tended to develop strategies through transformation of visual representations, while Subject 2 developed strategies through generalization of geometric concepts that had been understood. These findings indicate that solution strategies do not emerge spontaneously, but develop through the interaction between mathematical representations and students' geometric knowledge.

The synthesis of these two problem-solving patterns indicates that the strategy formation process begins when students construct an initial representation of the given problem. This representation then serves as the basis for exploring various possible solutions through the addition of auxiliary lines, the division of sides, and the use of geometric relationships. Each change in representation generates a different alternative strategy, while the chosen strategy encourages students to revise or enrich the representation they have constructed. Thus, mathematical representations and problem-solving strategies develop iteratively until a solution is found that is deemed most appropriate.

Based on these characteristics, this study reconstructs the Cognitive Model of Students' Strategies in Solving Open-Ended Geometry Problems as presented in Figure 4. This model consists of four main components, namely (1) understanding the problem through initial representation, (2) developing mathematical representation, (3) exploring and developing

solution strategies, and (4) verifying the suitability of strategies using geometric concepts. These four components do not occur linearly, but form a cycle that allows students to revise their representations and strategies until several valid solution alternatives are obtained.

The resulting model demonstrates that mathematical representations function not only as a communication medium but also as a means of thinking that facilitates the formation of solution strategies. Conversely, the solution strategies students choose also influence the representations they construct in subsequent stages. This reciprocal relationship is a key characteristic of the model developed in this study and distinguishes it from previous studies that primarily describe strategies or representations separately. Thus, this model provides a conceptual overview of how solution strategies develop through the interaction between mathematical representations and geometric knowledge in open-ended problem solving.

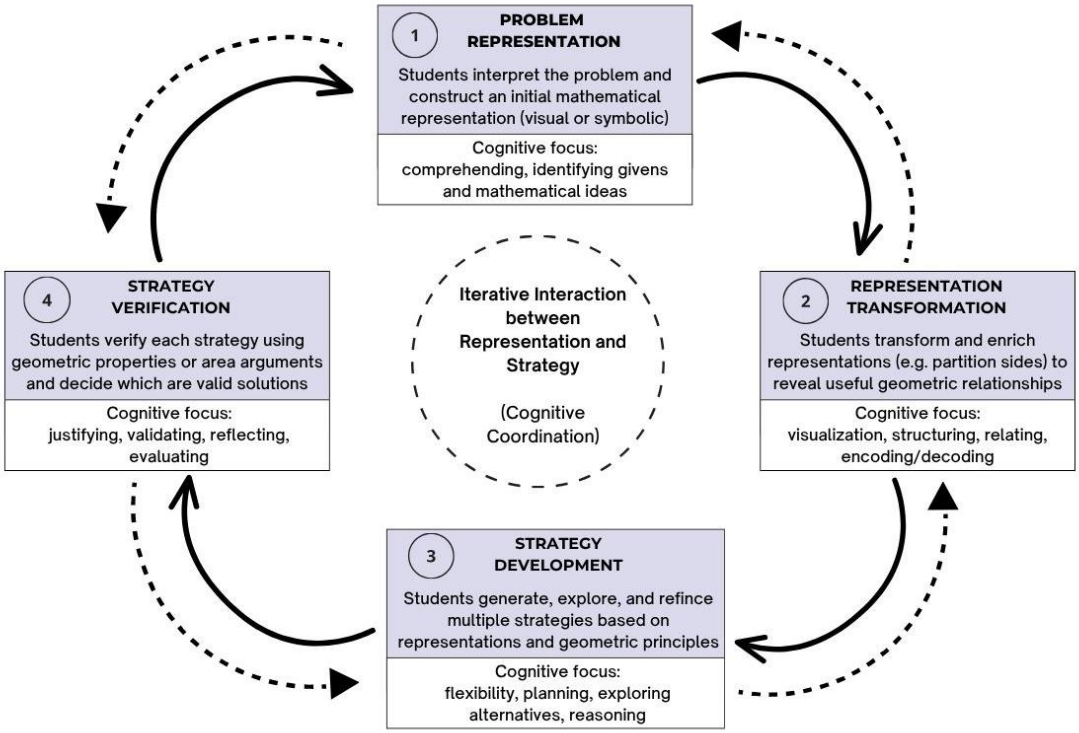


Figure 4. Cognitive Model of Students' Strategies in Solving Open-Ended Geometry Problems

Discussion

Finding on Strategy Development

The results of this study indicate that open-ended geometry problems provide students with the opportunity to generate diverse representations and solution strategies. Both subjects were able to generate more than one valid strategy, but exhibited different patterns of strategy

development. This finding reinforces the view that open-ended problems and multiple-solution tasks enable students to use mathematical knowledge more flexibly, explore alternative solutions, and demonstrate a variety of ways of constructing solutions (Levav-Waynberg & Leikin, 2012; Munroe, 2015). In the context of geometry, this diversity is important because the solution depends not only on the final result, but also on the student's ability to see geometric relationships that can be used to generate alternative solutions.

Mathematical representations play a crucial role in the development of strategies in both subjects. Drawings, auxiliary lines, similarity signs, and side divisions not only serve to present answers but also provide a structure that allows geometric relationships to be recognized and used in solving problems. This is consistent with the view that mathematical activity relies heavily on the use and transformation of semiotic representations (Duval, 2006). At the same time, the use of different representations allows students to explore alternative solutions and expands flexibility in solving problems (Levav-Waynberg & Leikin, 2012; Schindler & Lilienthal, 2020).

However, the two subjects showed different patterns. Subject 1 tended to develop strategies through representational transformation, while Subject 2 developed strategies more through the utilization and generalization of geometric principles across different configurations. This difference suggests that diverse solutions do not always develop through the same process. A strategy can develop because students change how they represent a problem, but it can also develop because students reuse geometric knowledge across different representations. This finding aligns with research on multiple-solution tasks, which suggests that flexibility in solving problems is related to the ability to use diverse knowledge and strategies and to see the connections between solutions (Levav-Waynberg & Leikin, 2012; Schindler & Lilienthal, 2020).

Intrepretation of The Cognitive Strategy Model

Based on these two patterns, the Student Cognitive Strategy Model reconstructed in this study positions problem representation, representation transformation, strategy development, and strategy verification as four components that interact iteratively. The contribution of this model is not to replace or broadly extend Duval's theory of semiotic representation or Schoenfeld's problem-solving framework, but rather to analytically link these two perspectives within the context of solving open-ended geometry problems. Duval's (2006) perspective provides a basis for understanding how mathematical representations are formed and transformed, while Schoenfeld's (1985) framework offers a foundation for understanding the

use of knowledge resources, strategies, and evaluation in problem-solving. The model reconstructed in this study connects these perspectives by demonstrating how changes in geometric representation, as evidenced in written solutions, relate to the development and verification of alternative solution strategies. Thus, representation and strategy are not treated as separate dimensions of analysis, but as components that are iteratively interconnected during the process of strategy formation.

The additional analytical value of this model lies in the identification of two distinct strategy-formation patterns within students' written solutions. The first pattern is the ‘representation-driven strategy’, involving the development of alternative strategies primarily through the transformation of visual configurations, while the second is the ‘concept-driven strategy’, involving the development of alternative strategies through the reuse and generalization of geometric principles across different configurations. These findings suggest that multiple strategies may be reconstructed through different pathways: one associated predominantly with changes in how the problem is represented and another associated with the reorganization and application of relevant geometric knowledge. Importantly, these patterns are not intended to classify students into fixed cognitive types, but rather to provide an analytical lens for understanding variations in strategy development. In this sense, the reconstructed model complements previous research on representation, problem-solving, and multiple-solution strategies by explicitly connecting representational changes with the development and verification of alternative strategies (Duval, 2006; Levav-Waynberg & Leikin, 2012; Munroe, 2015; Schindler & Lilienthal, 2020; Schoenfeld, 1985). However, since the model is reconstructed from written solutions, these two patterns are positioned as the outcomes of an analytical reconstruction based on observable artifacts, rather than as a directly observed sequence of students' mental processes.

Instructional Implications

These two patterns also have implications for mathematics teaching and the design of open-ended geometry tasks. Providing students with a task that allows multiple solutions may not, by itself, be sufficient to promote diverse strategy development. Teachers can support representation-driven strategy development by encouraging students to modify diagrams, add auxiliary lines, reorganize geometric configurations, or construct alternative representations of the same problem. In contrast, concept-driven strategy development can be supported by prompting students to identify the geometric principles underlying a solution and examine whether these principles can be reapplied or generalized to different configurations. Open-

ended geometry tasks can therefore be designed not only to allow multiple solutions, but also to deliberately create opportunities for representational transformation, conceptual generalization, comparison of alternative strategies, and mathematical verification. Such instructional support may provide students with multiple pathways for developing flexible problem-solving strategies rather than merely increasing the number of solutions they produce.

CONCLUSION

This study shows that solving open-ended geometry problems not only yields diverse answers but also reveals distinct patterns in the development of students' strategies. Analysis of the written solutions of two subjects shows that mathematical representations and solution strategies are dynamically interrelated. Subject 1 tends to develop strategies by transforming visual representations, while Subject 2 tends to develop strategies more through the application and generalization of geometric principles across different configurations. Based on these two patterns, a Cognitive Model of Student Strategies is reconstructed consisting of four components: problem representation, representation transformation, strategy development, and strategy verification. These four components interact iteratively, so that strategies are not seen as stand-alone end results, but as the result of the relationship between how students represent problems, develop alternative solutions, and verify the validity of solutions.

This study has several limitations that should be considered when interpreting the proposed Cognitive Strategy Model. First, the model was reconstructed from two purposively selected information-rich cases drawn from a single open-ended geometry task. Therefore, the model is intended to provide an in-depth analytical account of patterns identified in these cases rather than to represent a general model applicable to all students or all geometry problems. Second, the primary evidence consisted of students' written solutions. Although these artifacts provide observable traces of representation, strategy variation, and written verification, they do not provide direct access to students' internal cognitive processes or the exact temporal sequence in which these processes occurred. Consequently, the relationships among problem representation, representation transformation, strategy development, and strategy verification should be interpreted as analytically reconstructed patterns rather than universally applicable cognitive stages. Further research involving a wider range of participants, diverse open-ended geometry tasks, and complementary process-oriented methods such as think-aloud protocols, task-based interviews, or stimulated recall is needed to examine the model's transferability and explanatory robustness.

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